Combining Extension-based Semantics and Ranking-based Semantics for Abstract Argumentation

Elise Bonzon
LIPADE
Université Paris Descartes, France
elise.bonzon@mi.parisdescartes.fr

Jérôme Delobelle   **Sébastien Konieczny**
CRIL, CNRS
Université d'Artois, France
{delobelle,konieczny}@cril.fr

Nicolas Maudet
LIP6, CNRS
Sorbonne Université, 75005 Paris, France
nicolas.maudet@lip6.fr

Abstract

Two kinds of semantics exist for abstract argumentation. Extension-based semantics evaluate the acceptability of sets of arguments, while ranking-based semantics evaluate the strength of each argument. They focus on different aspects of the information conveyed by argumentation systems. After discussing pros and cons of both approaches, we study how to combine them, in order to take benefits from both. We propose six new families of semantics for abstract argumentation combining extension-based and ranking-based semantics. More precisely we propose to refine the ranking-based semantics using information coming from extension-based semantics acceptability of arguments, and to modify the extensions chosen by extension-based semantics using preferential information coming from ranking-based semantics.

Introduction

Argumentation is the process of confronting conflicting arguments. In the abstract argumentation framework (Dung 1995), the classical semantics are extension-based semantics. These semantics aim at evaluating which sets of arguments can be accepted together. These extensions are usually based on the conflict-freeness principle (two arguments in an extension can not attack each other) and on the self-defense principle (an extension has to defend each of its attacked argument). Thus, these semantics evaluate sets of arguments in a binary way (sets of arguments are or are not extensions for a given semantics).

In (Caminada 2006), labelling-based semantics have been introduced to associate different labellings to the arguments of any argumentation framework. A labelling is a function that maps each argument to the set $\{in, out, undec\}$, where in means that the argument is accepted for the labelling, out means that the argument is rejected, and $undec$ means that the argument is undecided. So these semantics still perform an evaluation of sets of arguments, just like extension-based semantics. And it has been shown that all extension-based semantics correspond to some labelling-based semantics.

More recently, it has been argued that this binary or ternary evaluation can be too rough for some applications, for example for online debate platforms (Leite and

Martins 2011), and the need of a more focused evaluation of each argument has been put forward. This led to the idea of ranking-based semantics (see e.g. (Cayrol and Lagasquie-Schiex 2005; Amgoud and Ben-Naim 2013; Grossi and Modgil 2015; Pu et al. 2015; Amgoud et al. 2016; Patkos, Bikakis, and Flouris 2016; Bonzon et al. 2016b)), where the aim is to (comparatively) evaluate each argument in an argumentation system. Ranking-based semantics are functions that map each argumentation framework to a ranking (usually a total pre-order) on its arguments. This ranking represents the comparative strength of each argument. Thus, conversely to extension-based (and labelling-based) semantics, this approach does not evaluate sets of arguments but each argument individually, based on its situation in the argumentation graph. A related kind of semantics are grading-based semantics (see e.g. (Besnard and Hunter 2001; Matt and Toni 2008; Leite and Martins 2011; da Costa Pereira, Tettamanzi, and Villata 2011)), where a numerical value is assigned to each argument. The evaluation is numerical instead of ordinal, but the aim is still to evaluate each argument individually. Clearly, if one defines a grading-based semantics, then this straightforwardly induces a corresponding ranking-based semantics.

Thus we may opt for two kinds of evaluations of arguments: at the level of set of arguments (with extension-based or labelling-based semantics) or at the level of single arguments (with ranking-based or grading-based semantics). These two ways to evaluate the information encoded in an argumentation framework are interesting, and are useful for different applications. The second approach is much more recent and more work is needed to better understand the notions and look for meaningful new semantics. But even the first kind of evaluation, although studied for a long time, still need some work for understanding their underlying principles (it worths mentioning that conversely to other reasoning tasks like inference (Kraus, Lehmann, and Magidor 1990; Makinson 1994) or revision (Alchourrón, Gärdenfors, and Makinson 1985; Gärdenfors 1988; Katsuno and Mendelzon 1991), there are no postulates for characterizing rational argumentation semantics and no representation theorem).

The starting point of this work is the observation that these two kinds of evaluation are in a sense orthogonal. They both can be used to extract some information about the status/strength/situation of (sets of) arguments. Instead of see-

ing these approaches as mutually exclusive, one natural idea is to try to take the best of both worlds and combine them. We believe that studying the potential of such a combination, as we initiate in this work, can be very fruitful for developing argumentation semantics.

In this work we propose six new families of semantics for abstract argumentation combining extension-based semantics and ranking-based semantics. More precisely, we propose to refine ranking-based semantics using information coming from extension-based semantics acceptability of arguments, and to modify the extensions chosen by extension-based semantics using preferential information coming from ranking-based semantics.

More precisely in the next section we will discuss the differences between the evaluation of arguments obtained by extension-based semantics and ranking-based semantics. We will then recall the necessary background on abstract argumentation. We will next show how to modify ranking-based semantics by taking into account information coming from extension-based semantics. We propose four ways to do that. The first one is by focusing only on the acceptability status of each argument (given by the extension-based semantics). The second one is based on a more precise evaluation of the acceptability status of each argument from (Wu and Caminada 2010). The third and fourth ones are modifications of a particular ranking-based method, the Propagation method (Bonzon et al. 2016b), where we allow a more fined-gained distinction of arguments using these acceptability status. Concerning the other way, i.e. how to modify extension-based semantics using ranking-based semantics, we show first that ranking-based semantics can be used to evaluate the extensions, and to select only the best of them. Then we discuss the possibility to take the ranks given by ranking-based semantics as a preferential information in a preference-based argumentation framework (Amgoud and Cayrol 2002) in order to select only the most convincing attacks.

Extension-based vs. Ranking-based semantics

Extension-based semantics (Dung 1995) are closely related to models of logic programs so they exhibit an all-or-nothing evaluation of sets of arguments. Amgoud and Ben-Naim (2013) underlines some characteristics specific to extension-based semantics: *Killing*: The impact of an attack from an argument y to an argument x is drastic, that is, if y belongs to an extension, then x is automatically excluded from that extension; *Existence*: One successful attack against an argument x has the same effect on an argument as any number of successful attacks. Indeed, one such attack is sufficient to "kill" x, several attacks cannot kill x to a greater extent; *Absoluteness*: The three possible statuses of the arguments (accepted, rejected or undecided) are absolute, that is, they make sense even without comparing them with each other; *Flatness*: All the arguments with the same status have the same level of acceptability. For example, all the accepted (respectively rejected) argument cannot be distinguished, *i.e* no accepted argument are more acceptable than another accepted argument. This kind of evaluation can be useful to define arguments from logical formulas. Here, the killing

and existence consideration seem essential to capture the fact that one attack is lethal and prevent any contradiction between arguments and thus obtain a consistent set of formulas.

However, in other applications, some of these properties can be discussed. Recently, online debate platforms are emerging on the internet. On these debate platforms, agents argue for or against a particular topic (in the form of a question or an affirmation) or other existing arguments. Often, the goal is not to find the arguments which can be accepted together but to evaluate how accepted is the question/affirmation. But more generally, when one faces many arguments, having a more detailed evaluation of arguments than the binary accepted/rejected obtained with extension-based semantics may be useful. Leite and Martins (2011) emphasize the limitations of classical acceptability semantics for this kind of applications. In addition, to accurately represent the opinions of thousands of users, it could be more appropriate to evaluate arguments using degrees of acceptability or gradual acceptability. With ranking-based semantics, we can precisely obtain a very detailed evaluation of the strength of each argument. This can be useful for these debate platforms, but also to select best arguments in all kinds of debates (persuasion, deliberation, etc.).

However we can see as a drawback the fact that the evaluation of each argument is not linked at all with its acceptance status: being an argument with a good evaluation does not mean that this argument should be accepted (under extension-based semantics), and even if we define "acceptance" with respect to the ranking, there are no natural threshold to make a distinction between accepted and non-accepted argument. Defining a ranking-based semantics that is compatible with the acceptance status of an extension-based semantics would be a solution. So we propose to build this kind of semantics by refining ranking-based semantics using extension-based semantics.

Conversely a drawback of extension-based semantics is that they do not allow a very detailed evaluation of arguments. It is for instance impossible to give a better evaluation to an unattacked argument than to all the arguments that this argument defends, whereas the acceptability of the latter depends on the acceptability of the unattacked argument. So one can use the detailed evaluation of arguments in order to modify extension-based semantics, for instance by selecting only the best extensions with respect to this evaluation.

We explore the two paths in the following.

Background Notions

In this section, we briefly recall some key elements of abstract argumentation frameworks.

Definition 1 *An* **argumentation framework (AF)** *is a pair* $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ *with* $\mathcal{A}$ *a finite set of arguments and* $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$ *is the* **attack relation** *between arguments. A set of arguments* $\mathcal{S} \subseteq \mathcal{A}$ *attacks an argument* $y \in \mathcal{A}$, *if there exists* $x \in \mathcal{S}$, *such that* $(x, y) \in \mathcal{R}$. $\mathcal{S}$ *defends* $z \in \mathcal{A}$ *against its attacker* y *if* $\mathcal{S}$ *attacks* y.

Abstract argumentation frameworks can be represented by directed graphs, where the nodes represent the arguments

and the edges represent the attack relation between two arguments. Let us now introduce some useful notions.

Definition 2 (Path) *Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework and $x, y \in \mathcal{A}$. A **path** P from y to x, noted $P(y, x)$, is a sequence $\langle x_0, \ldots, x_n \rangle$ of arguments such that $x_0 = x$, $x_n = y$ and $\forall i < n, (x_{i+1}, x_i) \in \mathcal{R}$. The length of the path P is n (the number of attacks it is composed of) and is denoted by $l_P = n$.*

Depending on the length of a path between two arguments, the argument at the beginning of this path can be an attacker and/or a defender (*i.e.*, an argument which attacks an attacker) of the argument at the end of the path.

Definition 3 (Defender/Attacker) *A **defender** (resp. **attacker**) of x is an argument situated at the beginning of an even-length (resp. odd-length) path. Let $R_n(x) = \{y \mid \exists P(y, x) \text{ with } l_P = n\}$ be the multiset of arguments that are bound by a path of length n to the argument x. Thus, an argument $y \in R_n(x)$ is a direct attacker of x if $n = 1$ or a direct defender of x if $n = 2$.*

Extension/Labelling-based semantics

In Dung's framework (Dung 1995), several acceptability semantics have been defined to select sets of arguments, called extensions, which can be conjointly accepted (*w.r.t* some criteria depending on the chosen semantic) for a given argumentation framework.

Definition 4 *Given an argumentation framework $AF = \langle \mathcal{A}, \mathcal{R} \rangle$. A set of arguments $\mathcal{S} \subseteq \mathcal{A}$ is conflict-free in AF if $\forall x, y \in \mathcal{S}, (x, y) \notin \mathcal{R}$. A conflict-free set $\mathcal{S}$ is admissible if it defends all its arguments against each of their direct attackers. An admissible set $\mathcal{S}$ is:*

- *a complete extension if each argument defended by $\mathcal{S}$ belongs to $\mathcal{S}$;*
- *a preferred extension if it is a $\subseteq$-maximal admissible set of AF;*
- *a stable extension if it attacks each argument in $\mathcal{A} \backslash \mathcal{S}$;*
- *the single grounded extension if it is the $\subseteq$-minimal complete extension of AF.*

*We denote by $\mathcal{E}_\sigma(AF)$ the set of extensions of AF for the semantics $\sigma \in \{$**co**(mplete), **pr**(eferred), **st**(able), **gr**(ounded)$\}$.*

An alternative way to represent the concepts of admissibility, as well as Dung's semantics, is by using a labelling-based approach (Caminada 2006).

Definition 5 (Labelling) *A labelling of an argumentation framework $\langle \mathcal{A}, \mathcal{R} \rangle$ is a function $\mathcal{L} : \mathcal{A} \rightarrow \{in, out, undec\}$. Given a label $l \in \{in, out, undec\}$, we define $l(\mathcal{L}) = \{x \in \mathcal{A} \mid \mathcal{L}(x) = l\}$.*

The notion of reinstatement labelling ensures that the mapping takes the attack relation into account.

Definition 6 (Reinstatement Labelling)
Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework. A labelling $\mathcal{L}$ is a reinstatement labelling of AF iff

- $\forall x \in \mathcal{A}, \mathcal{L}(x) = in$ iff $\forall y \in R_1(x), \mathcal{L}(y) = out$;
- $\forall x \in \mathcal{A}, \mathcal{L}(x) = out$ iff $\exists y \in R_1(x), \mathcal{L}(y) = in$;

- $\forall x \in \mathcal{A}, \mathcal{L}(x) = undec$ iff $\nexists y \in R_1(x), \mathcal{L}(y) = in$ and $\exists z \in R_1(x), \mathcal{L}(z) = undec$.

Definition 7 *Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework. A reinstatement labelling $\mathcal{L}$ is:*

- *a complete labelling;*
- *a grounded labelling if $in(\mathcal{L})$ is minimal (w.r.t. $\subseteq$);*
- *a preferred labelling if $in(\mathcal{L})$ is maximal (w.r.t. $\subseteq$);*
- *a stable labelling if $undec(\mathcal{L}) = \emptyset$.*

We denote by $\mathcal{L}_\sigma(AF)$ the set of reinstatement labellings of AF for the semantics $\sigma \in \{co, pr, st, gr\}$.

For an argumentation framework AF with at least one extension (resp. reinstatement labelling), we say that an argument is skeptically accepted if it belongs to all of AF's extensions (resp. it is labelled in in all of AF's reinstatement labellings). An argument is credulously accepted if it belongs to at least one of AF's extensions (resp. it is labelled in in at least one of AF's reinstatement labellings). Given a semantics σ, we denote by $sa_\sigma(AF)$ (resp. $ca_\sigma(AF)$) the set of skeptically (resp. credulously) accepted arguments in AF.

A more fine-grained notion of a justification status has also been introduced in (Wu and Caminada 2010) with a labelling-based justification status of the arguments in an argumentation framework. Concretely, the justification status of an argument consists of the set of labels that could reasonably be assigned to the argument w.r.t. the complete semantics.

Definition 8 (Justification status) *Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework and $x \in \mathcal{A}$. The justification status of x is the outcome yielded by the function $\mathcal{JS} : \mathcal{A} \rightarrow 2^{\{in, out, undec\}}$ such that $\mathcal{JS}(x) = \{\mathcal{L}(x) \mid \mathcal{L} \in \mathcal{L}_{co}(AF)\}$.*

For example, if an argument is labelled either in or $undec$ in all the complete labellings then the justification status of this argument is $\{in, undec\}$. Thus, there are 6 possible statuses to be considered: $\{in\}$, $\{out\}$, $\{undec\}$, $\{in, undec\}$, $\{out, undec\}$ and $\{in, out, undec\}$.

Ranking-based semantics

A ranking-based semantics allows to rank-order the arguments from the most to the least acceptable ones.

Definition 9 *A **ranking-based semantics** σ associates to any argumentation framework $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ a ranking $\succeq_{AF}^\sigma$ on $\mathcal{A}$, where $\succeq_{AF}^\sigma$ is a preorder (a reflexive and transitive relation) on $\mathcal{A}$. $x \succeq_{AF}^\sigma y$ means that x is at least as acceptable as y ($x \simeq_{AF}^\sigma y$ is a shortcut for $x \succeq_{AF}^\sigma y$ and $y \succeq_{AF}^\sigma x$, and $x \succ_{AF}^\sigma y$ is a shortcut for $x \succeq_{AF}^\sigma y$ and $y \nsucceq_{AF}^\sigma x$).*

A lot of these semantics were proposed (see *e.g.* (Cayrol and Lagasquie-Schiex 2005; Amgoud and Ben-Naim 2013; Grossi and Modgil 2015; Pu et al. 2015; Amgoud et al. 2016; Bonzon et al. 2016b)) with, for each of them, different behaviour and logical properties. In this work, we will focus on the categoriser-based ranking semantics to illustrate our method. This semantics has been initially introduced in (Besnard and Hunter 2001) and defined as a ranking-based semantics in (Pu et al. 2014).

Definition 10 *Let* $\langle \mathcal{A}, \mathcal{R} \rangle$ *be an argumentation framework. The categoriser function* $Cat : \mathcal{A} \to \;]0,1]$ *is defined as* $\forall x \in \mathcal{A}$,

$$Cat(x) = \begin{cases} 1 & \text{if } R_1(x) = \emptyset \\ \frac{1}{1+\sum_{y \in R_1(x)} Cat(y)} & \text{otherwise} \end{cases}$$

Definition 11 *The* **categoriser-based ranking semantics (Cat)** *associates to any* $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ *a ranking* $\succeq_{AF}^{Cat}$ *on* $\mathcal{A}$ *such that* $\forall x, y \in \mathcal{A}$, $x \succeq_{AF}^{Cat} y$ *iff* $Cat(x) \geq Cat(y)$.

Example 1 *Let us compute the set of extensions and the reinstatement labellings for* $\sigma \in \{co, pr, st, gr\}$ *and the ranking returned by the categoriser-based ranking semantics for the AF depicted in Figure 1.*
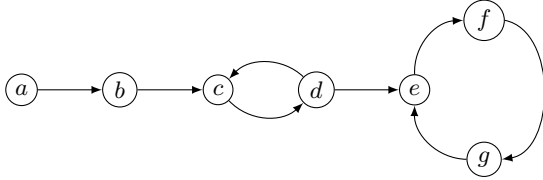

Figure 1: An argumentation framework AF

$\mathcal{E}_{gr}(AF) = \{a\}$
$\mathcal{E}_{pr}(AF) = \{\{a, c\}, \{a, d, f\}\}$
$\mathcal{E}_{st}(AF) = \{\{a, d, f\}\}$
$\mathcal{E}_{co}(AF) = \{\{a\}, \{a, c\}, \{a, d, f\}\}$

AF has three reinstatement labellings $\mathcal{L}_1$, $\mathcal{L}_2$ *and* $\mathcal{L}_3$ *with:*
$in(\mathcal{L}_1) = \{a\}, out(\mathcal{L}_1) = \{b\}, undec(\mathcal{L}_1) = \{c, d, e, f, g\}$
$in(\mathcal{L}_2) = \{a, c\}, out(\mathcal{L}_2) = \{b, d\}, undec(\mathcal{L}_2) = \{e, f, g\}$
$in(\mathcal{L}_3) = \{a, d, f\}, out(\mathcal{L}_3) = \{b, c, e, g\}, undec(\mathcal{L}_3) = \emptyset$

$$Cat(AF) = a \succ_{AF}^{Cat} f \succ_{AF}^{Cat} d \succ_{AF}^{Cat} g \succ_{AF}^{Cat} b \succ_{AF}^{Cat} c \succ_{AF}^{Cat} e$$

Many properties have been introduced in the literature (see (Bonzon et al. 2016a; Baroni, Rago, and Toni 2018) for an overview) aiming to better understand the behavior of these ranking-based semantics in various situations. Below we study how some of our methods stand with respect to these properties. We give their informal definition and point the reader to (Bonzon et al. 2016a) for the complete versions. Basic general properties are the fact that a ranking on a set of arguments should only depend on the attack relation (*Abstraction*, **Abs**); that the ranking between two arguments should be independent of arguments that are not connected to either of them (*Independence*, **In**); that all arguments can be compared (*Total*, **Tot**); and that all non-attacked arguments should be equally acceptable (*Non-attacked Equivalence*, **NaE**).

Local properties confine themselves to the level of direct attackers or direct defenders: (*Void Precedence*, **VP**) states that a non-attacked argument should be strictly more acceptable than any attacked argument; (*Self-Contradiction*, **SC**) states that an argument that attacks itself should be strictly less acceptable than an argument that does not; (*Cardinality Precedence*, **CP**) says that if an argument a has strictly more direct attackers than an other argument b, then b should be strictly more acceptable than a; (*Quality Precedence*, **QP**) says that if a has a direct attacker strictly more acceptable than any direct attacker of b, then a should be strictly more acceptable than b; (*Defense Precedence*, **DP**) states that for two arguments with the same number of direct attackers, a defended argument should be strictly more acceptable than a non-defended argument; (*Distributed-Defense Precedence*, **DDP**) considers that a defense where each defender attacks a distinct attacker is better than any other; (*Counter-Transitivity*, **CT**) states that if the direct attackers of b are (i) at least as numerous and (ii) as acceptable as those of a, then a should be at least as acceptable as b, while in its strict version (**SCT**) either (i) or (ii) must be strict, implying a strict comparison between a and b.

Global properties specify how the ranking should be affected on the basis of the comparison of attack and defense branches. More precisely: adding a defense branch to any argument should increase its acceptability (*Strict Addition of Defense Branch*, $\oplus$**DB**); the same properties have been defined but only when a defense branch is added to an attacked argument (*Addition of Defense Branch*, **+DB**); increasing the length of an attack branch of an argument should increase its acceptability (*Increase of Attack Branch*, $\uparrow$**AB**); adding an attack branch to an argument should decrease its acceptability (*Addition of Attack Branch*, **+AB**); and increasing the length of a defense branch of an argument should decrease its acceptability (*Increase of Defense Branch*, $\uparrow$**DB**). Note that +DB is indeed restricted to *attacked arguments*, otherwise its incompatibility with VP is obvious. In the same spirit, (*Attack vs Full Defense*, **AvsFD**) requires that an argument with only defense branches and no attack branch should be strictly more acceptable than an argument attacked once by a non-attacked argument.

Note that all these properties can not be satisfied together (Bonzon et al. 2016a), but checking which ones are satisfied by a semantics allow to characterize its behaviour.

Improving Ranking-based semantics using Extension-based semantics

Refining ranking-based semantics using acceptance status

The idea here is to constrain the rankings to be compatible with the acceptance status of the arguments. We lexicographically combine a ranking denoting the acceptance status of the arguments given by an extension-based semantics and the ranking given by a ranking-based semantics.

Definition 12 *Let* $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ *be an argumentation framework. Let* $\succeq_{AF}^1$ *and* $\succeq_{AF}^2$ *be two rankings on* $\mathcal{A}$. *The (lexicographical) refinement of* $\succeq_{AF}^2$ *by* $\succeq_{AF}^1$ *gives a new ranking* $\succeq_{AF}^{1,2}$ *such that* $\forall x, y \in \mathcal{A}$,

$$x \succeq_{AF}^{1,2} y \text{ iff } (x \succ_{AF}^2 y) \text{ or } (x \simeq_{AF}^2 y \text{ and } x \succeq_{AF}^1 y)$$

The following definition allows to build a ranking from the acceptance status given by an extension-based semantics[1]: an argument skeptically accepted is more acceptable

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[1]Please note that, a priori, any extension-based semantics can

than an argument credulously accepted which is more acceptable than a rejected argument.

Definition 13 *Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework and $\sigma \in \{co, pr, st, gr\}$. Let $\succeq_{AF}^{\sigma}$ be a ranking on $\mathcal{A}$ such that $\forall x, y \in \mathcal{A}$, $x \succeq_{AF}^{\sigma} y$ iff one of the following conditions is satisfied:* **i)** *$x \in sa_{\sigma}(AF)$,* **ii)** *$x \in ca_{\sigma}(AF) \backslash sa_{\sigma}(AF)$ and $y \notin sa_{\sigma}(AF)$,* **iii)** *$x, y \notin ca_{\sigma}(AF)$*

Definition 14 (Acceptance-based ranking semantics) *Let σ_1 be a ranking-based semantics and $\sigma_2 \in \{co, pr, st, gr\}$. The **acceptance-based ranking semantics** $\mathrm{ARS}_{\sigma_1, \sigma_2}$ associates to any $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ a ranking $\succeq_{AF}^{\sigma_1, \sigma_2}$ on $\mathcal{A}$ which is the refinement of $\succeq_{AF}^{\sigma_2}$ by $\succeq_{AF}^{\sigma_1}$.*

Example 2 *Let us first compute the arguments skeptically and credulously accepted w.r.t. the complete semantics on the AF depicted in Figure 1: $sa_{co}(AF) = \{a\}$ and $ca_{co}(AF) = \{a, c, d, f\}$.*

$$a \succ_{AF}^{co} c \simeq_{AF}^{co} d \simeq_{AF}^{co} f \succ_{AF}^{co} b \simeq_{AF}^{co} e \simeq_{AF}^{co} g$$

Let us recall the ranking returned by the categoriser-based ranking semantics:

$$a \succ_{AF}^{Cat} f \succ_{AF}^{Cat} d \succ_{AF}^{Cat} g \succ_{AF}^{Cat} b \succ_{AF}^{Cat} c \succ_{AF}^{Cat} e$$

Thus, when we combine the two rankings, the refinement-based ranking semantics returns the following ranking:

$$a \succ_{AF}^{Cat, co} f \succ_{AF}^{Cat, co} d \succ_{AF}^{Cat, co} c \succ_{AF}^{Cat, co} g \succ_{AF}^{Cat, co} b \succ_{AF}^{Cat, co} e$$

One can see on this example that g has quite a good evaluation for the ranking-based semantics $\succ_{AF}^{Cat}$, whereas it is a rejected argument. In particular it has a better evaluation that c that is credulously accepted (for the complete semantics). The combined ranking-based semantics $\succ^{Cat, co}$ allows to force c to be better than g.

So now the question is to know whether these modifications change the "rationality" of the ranking-based semantics, i.e. do these combined semantics satisfy less logical properties than the original ranking-based semantics?

Proposition 1 *Let σ_1 be a ranking-based semantics and $\sigma_2 \in \{co, pr, st, gr\}$. Let α be any property among Abs, In, VP, DP, DDP, SC, $\oplus$DB, +DB, +AB, $\uparrow$AB, $\uparrow$DB, Tot, NaE. If σ_1 satisfies the property α, then the semantics $\mathrm{ARS}_{\sigma_1, \sigma_2}$ satisfies the property α. The semantics $\mathrm{ARS}_{\sigma_1, gr}$ and $\mathrm{ARS}_{\sigma_1, st}$ satisfy QP, CT and SCT. The semantics $\mathrm{ARS}_{\sigma_1, \sigma_2}$ satisfies the property AvsFD and does not satisfy CP.*

It is interesting to note that, except for AvsFD and CP, the semantics satisfies the property if the original ranking-based semantics satisfies the properties. Thus, the compliance of the ranking-based semantics with respect to these properties is preserved using the refinement with the extension-based semantics. Better than that, it allows the enforcement of AvsFD that few semantics satisfy (Bonzon et al. 2016a). So it is an easy way to obtain new semantics satisfying AvsFD from standard semantics from the literature.

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be used in our method but for our properties, we choose to only focus on the four classical semantics.

Refining ranking-based semantics using justification status

Instead of focusing on the acceptability status of the arguments, we can also build a ranking from the labelling-based justification status of the arguments, that offers a more fined-gained distinction of the arguments with respect to the labellings/extensions. However, the definition from (Wu and Caminada 2010) (see Definition 8) only concerns the complete semantics. It is why we propose to extend the definition to all Dung's semantics.

Definition 15 (Extended justification status)
Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework, $\sigma \in \{co, pr, st, gr\}$ and $x \in \mathcal{A}$. The extended justification status of x is the outcome yielded by the function $\mathcal{JS} : \mathcal{A} \to 2^{\{in, out, undec\}}$ s.t. $\mathcal{JS}_{\sigma}(x) = \{\mathcal{L}_{\sigma}(x) \mid \mathcal{L} \in \mathcal{L}_{\sigma}(AF)\}$.

In addition to the 6 statuses $\{in\}$, $\{out\}$, $\{undec\}$, $\{in, undec\}$, $\{out, undec\}$ and $\{in, out, undec\}$, we must add the status $\{in, out\}$, that could not appear for the complete semantics, but which may be obtained, for instance with the preferred semantics.

With the graph depicted in Figure 2, we include the status $\{in, out\}$ in the hierarchy of the justification statuses.
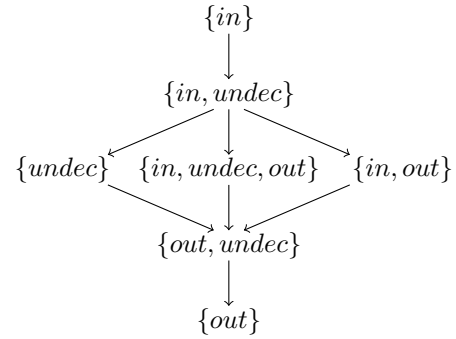
Figure 2: The hierarchy of the extended justification statuses

Thus, we can classify the statuses with the following ranking: $\{in\} \succ_{js} \{in, undec\} \succ_{js} \{undec\} \simeq \{in, out, undec\} \simeq \{in, out\} \succ_{js} \{out, undec\} \succ_{js} \{out\}$. According to this classification, we can say that an argument is more acceptable than another one if it has a better status.

Definition 16 *Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework and $\sigma \in \{co, pr, st, gr\}$. Let $\succeq_{AF}^{\mathcal{JS}_{\sigma}}$ be a ranking on $\mathcal{A}$ such that $\forall x, y \in \mathcal{A}$,*

$$x \succeq_{AF}^{\mathcal{JS}_{\sigma}} y \text{ iff } \mathcal{JS}_{\sigma}(x) \succeq_{js} \mathcal{JS}_{\sigma}(y)$$

Definition 17 (Justification-based ranking semantics) *Let σ_1 be a ranking-based semantics and $\sigma \in \{co, pr, st, gr\}$. The **justification-based ranking semantics** $\mathrm{JRS}_{\sigma_1, \sigma}$ associates to any $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ a ranking $\succeq_{AF}^{\sigma_1, \mathcal{JS}_{\sigma}}$ on $\mathcal{A}$ which is the refinement of $\succeq_{AF}^{\mathcal{JS}_{\sigma}}$ by $\succeq_{AF}^{\sigma_1}$.*

Example 3 *We recall that the argumentation framework depicted in Figure 1 has three complete labellings $\mathcal{L}_1$, $\mathcal{L}_2$ and*

$\mathcal{L}_3$. *From these different complete labellings, the labelling-based justification statuses of each argument in the AF is* $\mathcal{JS}_{co}(a) = \{in\}$, $\mathcal{JS}_{co}(b) = \{out\}$, $\mathcal{JS}_{co}(c) = \mathcal{JS}_{co}(d) = \{in, out, undec\}$, $\mathcal{JS}_{co}(e) = \mathcal{JS}_{co}(g) = \{undec, out\}$ *and* $\mathcal{JS}_{co}(f) = \{in, undec\}$. *So we obtain the following ranking:*

$$a \succ_{AF}^{JS_{co}} f \succ_{AF}^{JS_{co}} c \simeq_{AF}^{JS_{co}} d \succ_{AF}^{JS_{co}} e \simeq_{AF}^{JS_{co}} g \succ_{AF}^{JS_{co}} b$$

Combined with the ranking returned by the categoriser-based ranking semantics, we obtain the following ranking:

$$a \succ_{AF}^{Cat,JS_{co}} f \succ_{AF}^{Cat,JS_{co}} d \succ_{AF}^{Cat,JS_{co}} c \succ_{AF}^{Cat,JS_{co}} g \succ_{AF}^{Cat,JS_{co}} e \succ_{AF}^{Cat,JS_{co}} b$$

Proposition 2 *Let* σ_1 *be a ranking-based semantics and* $\sigma_2 \in \{co, pr, st, gr\}$. *Let* α *be any property among Abs, In, VP, DP, DDP, $\oplus$DB, +DB, +AB, $\uparrow$AB, $\uparrow$DB, Tot, NaE. If* σ_1 *satisfies the property* α, *then the semantics* $\mathrm{ARS}_{\sigma_1,JS_{\sigma_2}}$ *satisfies the property* α. *The semantics* $\mathrm{ARS}_{\sigma_1,JS_{gr}}$ *and* $\mathrm{ARS}_{\sigma_1,JS_{st}}$ *satisfy QP, CT and SCT. The semantics* $\mathrm{ARS}_{\sigma_1,JS_{\sigma_2}}$ *satisfies the property AvsFD and does not satisfy CP, SC.*

One can remark that the difference of properties satisfied between this semantics and the previous one is minor. Indeed, the only difference concerns the Self-Contradiction (SC) property and can be explained by the fact that an argument which attacks itself is labelled $undec$ if it is not attacked by other arguments. Thus, this argument is more acceptable than an argument directly attacked by a non-attacked argument while the skeptical and credulous inference functions always considers the two arguments as rejected.

Refining Propagation semantics using acceptance and justification status

We propose to adapt the propagation principle introduced in (Bonzon et al. 2016b). The idea of propagation is to give a better initial value to non-attacked arguments than to attacked arguments, in order to improve their impact in the evaluation of arguments. Then these values are propagated into the argumentation framework.

Similarly to the previous sections we propose here to use acceptance status and justification status to allow a more fine-grained initial evaluation.

Let us formally define the propagation principle.

Definition 18 (Propagation) *Let* $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ *be an argumentation framework. Let* $v : \mathcal{A} \to [0, 1]$ *be a valuation function giving an initial weight to each argument. The valuation* P *of* $x \in \mathcal{A}$, *at step* $i \in \mathbb{N}$, *is given by:*

$$P_i^v(x) = \begin{cases} v(x) & \text{if } i = 0 \\ P_{i-1}^v(x) + (-1)^i \sum_{y \in R_i(x)} v(y) & \text{otherwise} \end{cases}$$

*The **propagation vector** of* x *is denoted* $P^v(x) = \langle P_0^v(x), P_1^v(x), \dots \rangle$.

Like in (Bonzon et al. 2016b), we use the lexicographical order to compare the propagation vectors of each argument.

Definition 19 (Lexicographical order) *A lexicographical order between two vectors of real numbers* $V = \langle V_1, \dots, V_n \rangle$ *and* $V' = \langle V'_1, \dots, V'_n \rangle$ *is defined as follows:* $V \succ_{lex} V'$ *iff* $\exists i \le n$ *s.t.* $V_i > V'_i$ *and* $\forall j < i, V_j = V'_j$.

Definition 20 ($Propa^v$) *The ranking-based semantics* $Propa^v$ *associates to any argumentation framework* $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ *a ranking* $\succeq_{AF}^{P_v}$ *on* $\mathcal{A}$, *where* v *is a valuation function, such that* $\forall x, y \in \mathcal{A}$, $x \succeq_{AF}^{P_v} y$ *iff* $P^v(x) \succeq_{lex} P^v(y)$.

The ranking returned by the semantics clearly depends on the chosen valuation function v. In (Bonzon et al. 2016b) the valuation function takes only two values (one for non-attacked and one for attacked arguments). We will now propose more complex functions. Let us first define a valuation function which takes into account the level of acceptability of arguments.

Definition 21 ($v_{\vec{z}_\sigma}$) *Let* $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ *be an argumentation framework and* $\vec{z}_\sigma = \langle \alpha, \beta, \gamma, \delta \rangle$ *be a vector of real number linked to the semantics* $\sigma \in \{co, pr, st, gr\}$. *The valuation function* $v_\sigma : \mathcal{A} \to [0, 1]$ *is defined as* $\forall x \in \mathcal{A}$,

$$v_{\vec{z}_\sigma}(x) = \begin{cases} \alpha & \text{if } R_1(x) = \emptyset \\ \beta & \text{if } x \in sa_\sigma(AF) \text{ and } R_1(x) \ne \emptyset \\ \gamma & \text{if } x \in ca_\sigma(AF) \backslash sa_\sigma(AF) \\ \delta & \text{if } x \notin ca_\sigma(AF) \end{cases}$$

with $1 \ge \alpha > \beta > \gamma > \delta \ge 0$.

So a preference is given to non-attacked arguments, then to skeptically accepted arguments, then to credulously accepted ones, and the worst ones are rejected arguments.

Example 4 *Applying the complete semantics on the AF depicted in Figure 1 allows to say that* a *is the only argument which is not attacked while* c, d *and* f *are credulously (and not skeptically) accepted and* b, e *and* g *are considered as rejected.*
With $\vec{z}_{co} = \langle 1, 0.7, 0.3, 0 \rangle$, *we obtain the following table which sums up the valuation of each argument at each step:*

$P_i^{\vec{z}_{co}}$	a	b	c	d	e	f	g
0	*1*	*0*	*0.3*	*0.3*	*0*	*0.3*	*0*
1	*1*	*-1*	*0*	*0*	*-0.3*	*0.3*	*-0.3*
2	*1*	*-1*	*1.3*	*0.3*	*0.3*	*0.6*	*-0.3*

When we lexicographically compare the propagation vector of each argument, we obtain the following ranking:

$$a \succ_{AF}^{P_{\vec{z}_{co}}} f \succ_{AF}^{P_{\vec{z}_{co}}} c \succ_{AF}^{P_{\vec{z}_{co}}} d \succ_{AF}^{P_{\vec{z}_{co}}} e \succ_{AF}^{P_{\vec{z}_{co}}} g \succ_{AF}^{P_{\vec{z}_{co}}} b$$

Let us check which properties are satisfied:

Proposition 3 *Let* $\sigma \in \{co, pr, st, gr\}$. *The semantics* $Propa^{\vec{z}_\sigma}$ *satisfies Abs, In, VP, DP, $\uparrow$AB, $\uparrow$DB, +AB, Tot, NaE and AvsFD. The other properties are not satisfied.*

The set of properties satisfied by $Propa^{\vec{z}_\sigma}$ is close to the ones satisfied by the semantics $Propa_\epsilon$ introduced in (Bonzon et al. 2016b). The differences come from the AvsFD property that is satisfied by $Propa^{\vec{z}_\sigma}$ thanks to the distinction done between attacked arguments in the initial evaluation, and from the fact that SCT and CT are not satisfied.

We also define another valuation function by considering the extended justification status (see Definition 15).

Definition 22 ($v_{\mathbf{JS}_\sigma}$) *Let* $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ *be an argumentation framework and* $\vec{js}_\sigma = \langle \alpha, \beta, \gamma, \delta, \epsilon, \omega \rangle$ *be a vector of real number linked to the semantics* $\sigma \in \{co, pr, st, gr\}$. *The valuation function* $v_{\vec{js}_\sigma} : \mathcal{A} \to [0,1]$ *is defined as* $\forall x \in \mathcal{A}$,

$$v_{\vec{js}_\sigma}(x) = \begin{cases} \alpha & \text{if } R_1(x) = \emptyset \\ \beta & \text{if } \mathcal{JS}_\sigma(x) = \{in\} \text{ and } R_1(x) \neq \emptyset \\ \gamma & \text{if } \mathcal{JS}_\sigma(x) = \{in, undec\} \\ \delta & \text{if } \mathcal{JS}_\sigma(x) \in \{\{undec\}, \{in, out\}, \\ & \qquad \qquad \{in, undec, out\}\} \\ \epsilon & \text{if } \mathcal{JS}_\sigma(x) = \{undec, out\} \\ \omega & \text{if } \mathcal{JS}_\sigma(x) = \{out\} \end{cases}$$

with $1 \geq \alpha > \beta > \gamma > \delta > \epsilon > \omega \geq 0$.

Example 5 *Let us recall the justification status labelling of each argument:* $\mathcal{JS}_{co}(a) = \{in\}$, $\mathcal{JS}_{co}(b) = \{out\}$, $\mathcal{JS}_{co}(c) = \mathcal{JS}_{co}(d) = \{in, out, undec\}$, $\mathcal{JS}_{co}(e) = \mathcal{JS}_{co}(g) = \{undec, out\}$ *and* $\mathcal{JS}_{co}(f) = \{in, undec\}$. *So, with* $\vec{js}_{co} = \langle 1, 0.8, 0.6, 0.4, 0.2, 0 \rangle$, *we have:*

$P_i^{\vec{js}_{co}}$	a	b	c	d	e	f	g
0	*1*	*0*	*0.4*	*0.4*	*0.2*	*0.6*	*0.2*
1	*1*	*-1*	*0*	*0*	*-0.4*	*0.4*	*-0.4*
2	*1*	*-1*	*1.4*	*0.4*	*0.6*	*1*	*-0.2*

And, consequently, we obtain the following ranking:

$$a \succ_{AF}^{P_{\vec{js}_{co}}} f \succ_{AF}^{P_{\vec{js}_{co}}} c \succ_{AF}^{P_{\vec{js}_{co}}} d \succ_{AF}^{P_{\vec{js}_{co}}} e \succ_{AF}^{P_{\vec{js}_{co}}} g \succ_{AF}^{P_{\vec{js}_{co}}} b$$

As shown in the following proposition, increasing the number of distinctions between the attacked argument does not change the properties satisfied by the propagation semantics.
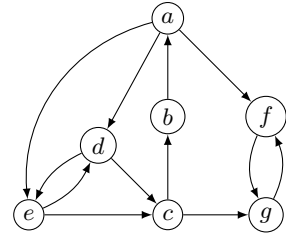
Proposition 4 *Let* $\sigma \in \{co, pr, st, gr\}$. *The semantics* $Propa^{z_{js_\sigma}}$ *satisfies Abs, In, VP, DP,* $\uparrow$*AB,* $\uparrow$*DB,* $+$*AB, Tot, NaE and AvsFD. The other properties are not satisfied.*

Improving Extension-based using Ranking-based semantics

In this section, we propose three different ways to use ranking-based semantics to modify the results obtained by extension-based semantics. The aim of the first two methods is to decrease the number of extensions, in order to allow more inferences, thanks to ranking-based semantics. The last method disregards the attacks that come from bad arguments with respect to the ranking-based semantics.

Select the best extensions

As shown with the argumentation framework depicted in Figure 3, when many cycles with even length exist, extension-based semantics return several extensions (except for the grounded semantics which always return a unique extension). As discussed in (Konieczny, Marquis, and Vesic 2015), in this case, selecting the arguments skeptically and credulously accepted can be problematic. Indeed, if there are many extensions, using skeptical inference can give almost no information and the credulous inference can give too many arguments. This is illustrated on the AF depicted

$\mathcal{E}_{gr}(AF) = \{\}$
$\mathcal{E}_{pr}(AF) = \{\{a,c\}, \{b,d,f\}, \{b,e,f\}, \{b,d,g\}, \{b,e,g\}\}$
$\mathcal{E}_{st}(AF) = \{\{a,c\}, \{b,d,f\}, \{b,e,f\}, \{b,d,g\}, \{b,e,g\}\}$
$\mathcal{E}_{co}(AF) = \{\emptyset, \{a,c\}, \{b,d,f\}, \{b,e,f\}, \{b,d,g\}, \\ \{b,e,g\}\}$

$$Cat(AF) = b \succ_{\text{AF}}^{\text{Cat}} a \succ_{\text{AF}}^{\text{Cat}} c \succ_{\text{AF}}^{\text{Cat}} g \succ_{\text{AF}}^{\text{Cat}} d \simeq_{\text{AF}}^{\text{Cat}} e \succ_{\text{AF}}^{\text{Cat}} f$$

Figure 3: An AF with many extensions

in Figure 3 because, when we focus on the preferred semantics (the remark also holds for the stable and complete semantics) the set of arguments skeptically accepted is empty, while the set of arguments credulously accepted contains all the arguments of AF. There exist works where additional information (*e.g.* weight on the attacks, preferences) are used to reduce the number of extensions in a given argumentation framework (*e.g.* (Coste-Marquis et al. 2012; Amgoud and Vesic 2014)). However, our goal is to reduce the set of extensions without any additional information in the argumentation framework. While, in (Konieczny, Marquis, and Vesic 2015), the attack relation is taken into account to discriminate some extensions, we propose here to consider the ranking returned by a ranking-based semantics to select the "best" extensions. For this purpose, we propose two approches.

Comparing the arguments' ranks The first criterion we consider is the rank that an argument has into a ranking of arguments returned by a ranking-based semantics. Indeed, suppose an agent selecting the most convincing arguments to put forward in a debate so as to convince an audience: the agent wants to use the best arguments (but still remain consistent and in a position to defend against possible attacks).

Definition 23 (Rank) *Let* $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ *be an argumentation framework. Given a ranking-based semantics* σ, *the rank of* $x \in \mathcal{A}$ *w.r.t.* $\succeq_{AF}^\sigma$, *denoted by* $r_\sigma(x)$, *is the level in which it belongs in the ordered sequence of equivalence classes of* $\mathcal{A}$ *with respect to* $\succeq_{AF}^\sigma$. *So basically* $r_\sigma(x) = i$ *where* i *is the longest path* $x_1 \succ_{AF}^\sigma \ldots \succ_{AF}^\sigma x_i \succ_{AF}^\sigma x$; *and* $r_\sigma(x) = 0$ *if* $\nexists y \in \mathcal{A}$ *s.t.* $y \succ_{AF}^\sigma x$.

Example 6 *If we consider the ranking returned by the categoriser semantics on* AF, *the rank of each argument is* $r_{Cat}(a) = 1$, $r_{Cat}(b) = 0$, $r_{Cat}(c) = 2$, $r_{Cat}(d) = 4$, $r_{Cat}(e) = 4$, $r_{Cat}(f) = 5$ *and* $r_{Cat}(g) = 3$.

Given a ranking-based semantics, the rank multiset of an extension obtained from an extension-based semantics is composed of the rank of each of its arguments.

Definition 24 (Rank multiset) *Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework, σ_1 be an extension-based semantics and σ_2 be a ranking-based semantics. For an extension $\mathcal{E} \in \mathcal{E}_{\sigma_1}(AF)$, with $\mathcal{E} = \{x_1, \ldots, x_n\}$, we define its rank multiset as $rv_{\sigma_2}(\mathcal{E}) = (r_{\sigma_2}(x_1), \ldots, r_{\sigma_2}(x_n))$.*

Example 6 (cont.) *Focusing on the preferred extension $\{b, d, f\}$ and the categoriser-based semantics, we have $rv_{Cat}(\{b, d, f\}) = (0, 4, 5)$.*

We use an aggregation function in order to aggregate the values belonging to the same rank multiset.

Definition 25 (Aggregation function) *We say that $\oplus$ is an aggregation function if for every $n \in \mathbb{N}$, $\oplus$ is a mapping from $\mathbb{N}^n$ to $\mathbb{N}$ such that:*

- *if $x_i \geq x_i'$, then*
  $\oplus(x_1, \ldots, x_i, \ldots, x_n) \geq \oplus(x_1, \ldots, x_i', \ldots, x_n)$
- *$\oplus(x_1, \ldots, x_n) = 0$ iff for every i, $x_i = 0$*
- *$\oplus(x) = x$.*

Many typical examples of aggregation functions exist such as $sum, max, min, leximax, leximin$, etc.

The goal is now to compare the score assigned to each extension in order to select the best ones with respect to the chosen criterion.

Definition 26 (Rank-based extensions) *Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework, σ_1 be an extension-based semantics, σ_2 be a ranking-based semantics and $\oplus$ be an aggregation function. The set of rank-based extensions (RBE) is defined as $\mathrm{RBE}_{\sigma_1, \sigma_2}^{\oplus}(AF) = \underset{\mathcal{E} \in \mathcal{E}_{\sigma_1}(AF)}{\mathrm{argmin}} \oplus(rv_{\sigma_2}(\mathcal{E}))$*

Obviously, the resulting set of extensions depends of the chosen aggregation function. Indeed, using the average favours the extensions with few arguments but which have a good rank (even if these are not the best ranks) while when the leximin is used, the number of arguments has no impact because the rank of the best argument in each extension is first compared and in case of tie for some extensions, we compare the second best rank and so on. Thus, an agent may prefer to use either lexicographical or average aggregators, depending on how she believes the audience will perceive arguments (either focus on the most significant, or asses the debate globally, etc.).

Example 7 *Let us select the best extensions among the set of preferred extensions of AF. The rank of each argument is computed on the basis of the categoriser-based ranking semantics. We only focus on the average and the leximin as aggregation function.*

		$\mathcal{L}eximin$	avg
$\mathcal{E}_1$	$\{a, c\}$	$(1, 2)$	1.5
$\mathcal{E}_2$	$\{b, d, f\}$	$(0, 4, 5)$	3
$\mathcal{E}_3$	$\{b, e, f\}$	$(0, 4, 5)$	3
$\mathcal{E}_4$	$\{b, d, g\}$	$(0, 3, 4)$	2.33
$\mathcal{E}_5$	$\{b, e, g\}$	$(0, 3, 4)$	2.33

Thus, for the leximin, we have $\mathrm{RBE}_{pr, Cat}^{\mathcal{L}eximin}(AF) = \{\mathcal{E}_4, \mathcal{E}_5\}$, while when the average is used, we have $\mathrm{RBE}_{pr, Cat}^{avg}(AF) = \{\mathcal{E}_1\}$.

Proposition 5 *For every $\oplus$, for every semantics σ_1 and σ_2, for every $AF = \langle \mathcal{A}, \mathcal{R} \rangle$, for every $x \in \mathcal{A}$,*

- $\mathrm{RBE}_{\sigma_1, \sigma_2}^{\oplus}(AF) \subseteq \mathcal{E}_{\sigma_1}(AF)$
- $sa_{\sigma_1}(AF) \subseteq \bigcup_{\mathcal{E} \in \mathrm{RBE}_{\sigma_1, \sigma_2}^{\oplus}(AF)} \mathcal{E} \subseteq ca_{\sigma_1}(AF)$

Baroni and Giacomin (2007) pointed out a set of properties for extension-based argumentation semantics.

Proposition 6 *Let $\oplus$ be an aggregation function and σ_2 be a ranking-based semantics. Let α be any property among I-maximality, Admissibility, Strong Admissibility, Reinstatement, Weak Reinstatement and CF-Reinstatement (Baroni and Giacomin 2007). If the semantics σ_1 satisfies the property α, then the semantics $\mathrm{RBE}_{\sigma_1, \sigma_2}^{\oplus}$ satisfies the property α.*

So RBE satisfies the same properties as the underlying extension-based (or labelling-based) semantics they are built from, with the exception of the directionality property, just like in (Konieczny, Marquis, and Vesic 2015).

Pairwise comparison Our second approach consists in comparing all pairs of extensions based on the number of arguments in one extension which are more acceptable than the arguments in another extension. Such choice of comparison could be interesting for example when the user may have the opportunity to come up with alternative extensions herself and ask for justification as to why this other extension was not picked.

Definition 27 *Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework. Let σ_1 be an extension-based semantics, σ_2 be a ranking-based semantics and $\mathcal{E}, \mathcal{E}' \in \mathcal{E}_{\sigma_1}(AF)$. We have*

$$\mathcal{N}_{\sigma_2}(\mathcal{E}, \mathcal{E}') = |\{(x, y) \ s.t. \ x \succ_{AF}^{\sigma_2} y \ \text{with} \ x \in \mathcal{E} \ \text{and} \ y \in \mathcal{E}'\}|$$

Example 8 *Let us consider the set of extensions returned by the preferred semantics and the ranking returned by the categoriser-based semantics. For example, $\mathcal{N}_{Cat}(\mathcal{E}_1, \mathcal{E}_4) = 4$ because $c \succ^{Cat} d$, $c \succ^{Cat} g$, $a \succ^{Cat} d$ and $a \succ^{Cat} g$. So, following the same reasoning, when we compare all the extensions, we obtain the following table:*

$\mathcal{N}_{Cat}$	$\mathcal{E}_1$	$\mathcal{E}_2$	$\mathcal{E}_3$	$\mathcal{E}_4$	$\mathcal{E}_5$
$\mathcal{E}_1 = \{a, c\}$	$\times$	4	4	4	4
$\mathcal{E}_2 = \{b, d, f\}$	2	$\times$	0	0	0
$\mathcal{E}_3 = \{b, e, f\}$	2	0	$\times$	0	0
$\mathcal{E}_4 = \{b, d, g\}$	2	1	1	$\times$	0
$\mathcal{E}_5 = \{b, e, g\}$	2	1	1	0	$\times$

The approach consists in counting how many extensions are defeated by a given extension and select the extension(s) which obtains the best score.

Definition 28 (Acceptability-based extensions)
Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework, σ_1 be an extension-based semantics and σ_2 be a ranking-based semantics. The set of acceptability-based extensions (ABE) is defined as follows:
$\mathrm{ABE}_{\sigma_1, \sigma_2}(AF) =$
$\quad \underset{\mathcal{E} \in \mathcal{E}_{\sigma_1}(AF)}{\mathrm{argmax}} |\{\mathcal{E}' \in \mathcal{E}_{\sigma_1}(AF) : \mathcal{N}_{\sigma_2}(\mathcal{E}, \mathcal{E}') > \mathcal{N}_{\sigma_2}(\mathcal{E}', \mathcal{E})\}|$

Example 8 (cont.) *We can see that the extension $\mathcal{E}_1$ defeats all the other extensions, so,* $\mathtt{ABE}_{pr,Cat}(AF) = \{\mathcal{E}_1\}$.

Proposition 7 *For every semantics σ_1 and σ_2, for every $AF = \langle \mathcal{A}, \mathcal{R} \rangle$, for every $x \in \mathcal{A}$,*

- $\mathtt{ABE}_{\sigma_1,\sigma_2}(AF) \subseteq \mathcal{E}_{\sigma_1}(AF)$
- $sa_{\sigma_1}(AF) \subseteq \bigcup_{\mathcal{E} \in \mathtt{ABE}_{\sigma_1,\sigma_2}(AF)} \mathcal{E} \subseteq ca_{\sigma_1}(AF)$

Although the obtained semantics are different from the RBE semantics, ABE semantics exhibit the same properties.

Proposition 8 *Let σ_2 be a ranking-based semantics. Let α be any property among I-maximality, Admissibility, Strong Admissibility, Reinstatement, Weak Reinstatement and CF-Reinstatement (Baroni and Giacomin 2007).*
If the semantics σ_1 satisfies the property α, then the semantics $\mathtt{ABE}_{\sigma_1,\sigma_2}$ satisfies the property α.

Removing attacks

As a last possible modification of extension-based semantics with ranking based-semantics we will look at a more drastic modification, where we put more emphasis on the ranking-based semantics. The idea here is to give strong priority to strong arguments with respect to the ranking-based semantics, by not considering attacks from weaker arguments to stronger ones. So we will use the preference-based argumentation framework of Amgoud and Cayrol (2002), by considering the ranking obtained by the ranking-based semantics as the preference relation between arguments.
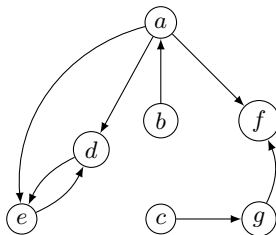
Amgoud and Cayrol (2002) redefine the attack relation in saying that an argument x defeats an argument y if and only if there exists an attack from x to y and y is not preferred to x with respect to the preference relation. And this is this defeat relation that is used to define extensions.

Definition 29 (AF_σ) *Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework and σ be a ranking-based semantics. An AF_σ is a triplet $\langle \mathcal{A}', \mathcal{R}', \succeq^\sigma_{AF} \rangle$ where:*

- $\mathcal{A}' = \mathcal{A}$
- $\mathcal{R}' = \{(a,b) \mid (a,b) \in \mathcal{R} \text{ and } a \succeq^\sigma_{AF} b\}$
- $\succeq^\sigma_{AF}$ *is the ranking on $\mathcal{A}$ returned by the ranking-based semantics σ from AF*

The acceptability of the arguments are then defined in the standard way from this new argumentation framework.

Example 9 *Let us focus on the argumentation framework depicted in Figure 3 and on the categoriser-based ranking semantics. Following the definition, we must remove the attacks (c,b) (because $b \succ^{Cat}_{AF} c$), (d,c) (because $c \succ^{Cat}_{AF} d$), (e,c) (because $c \succ^{Cat}_{AF} e$) and (f,g) (because $g \succ^{Cat}_{AF} f$). Thus, we obtain the following AF_{Cat} and its extensions:*

$\mathcal{E}_{gr}(AF_{Cat}) = \{b,c,f\}$
$\mathcal{E}_{pr}(AF_{Cat}) = \{\{b,c,e,f\},\{b,c,d,f\}\}$
$\mathcal{E}_{st}(AF_{Cat}) = \{\{b,c,e,f\},\{b,c,d,f\}\}$
$\mathcal{E}_{co}(AF_{Cat}) = \{\{b,c,f\},\{b,c,e,f\},\{b,c,d,f\}\}$

One can remark that the computed extensions are not necessarily conflict-free with respect to the original AF due to the removal of some attacks. That is perfectly natural since we consider that these attack are not legitimate ones, so conflict-freeness has to be considered with respect to the defeat relation, not the attack one.

But, in the case where one wants a conflict-free result with respect to the original argumentation framework, we propose now a method to "rationalize" these extensions by extracting conflict-free subsets.

For this purpose, for a given set of (potentially not conflict-free) arguments, we select the subsets of arguments (maximal w.r.t. $\subseteq$) which are conflict-free and which respect the constraint saying that when a conflict exists between two arguments, the most acceptable argument (w.r.t. the ranking-based semantics used to define AF_σ) is selected.

Definition 30 *Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework and σ be a ranking-based semantics. The function $\mathtt{CF}^\sigma_{AF} : \mathcal{A} \to 2^{\mathcal{A}}$ allows to compute the conflict-free sets of arguments from $\mathcal{X} \subseteq \mathcal{A}$:*

$$\mathtt{CF}^\sigma_{AF}(\mathcal{X}) = \{\mathcal{X}_1, \ldots, \mathcal{X}_n\}$$

where for each $\mathcal{X}_i$:

- $\mathcal{X}_i \subseteq \mathcal{X}$
- $\mathcal{X}_i$ *is maximal w.r.t. $\subseteq$*
- $\mathcal{X}_i$ *is conflict-free w.r.t. AF*
- *If $y \in \mathcal{X}$ and $y \notin \mathcal{X}_i$ then $\exists z \in \mathcal{X}_i$ s.t. $z \succeq^\sigma_{AF} y$ and $(y,z) \in \mathcal{R}$*

Please note that if $\mathcal{X}$ is conflict-free then $\mathtt{CF}^\sigma_{AF}(\mathcal{X}) = \mathcal{X}$.

We are now able to define how to make conflict-free (w.r.t. an given AF) the set of extensions computed from AF_σ.

Definition 31 *Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework. Let σ_1 be an extension-based semantics and σ_2 be a ranking-based semantics. We have*

$$\mathcal{E}^*_{\sigma_1,\sigma_2}(AF) = \{ \mathtt{CF}^{\sigma_2}_{AF}(\mathcal{E}) \mid \mathcal{E} \in \mathcal{E}_{\sigma_1}(AF_{\sigma_2})\}$$

Example 9 (cont.) *According to the previous definition, we obtain the following extended extensions:*
$\mathcal{E}^*_{gr,Cat}(AF) = \{b,f\}$
$\mathcal{E}^*_{pr,Cat}(AF) = \{\{b,e,f\},\{b,d,f\}\}$
$\mathcal{E}^*_{st,Cat}(AF) = \{\{b,e,f\},\{b,d,f\}\}$
$\mathcal{E}^*_{co,Cat}(AF) = \{\{b,f\},\{b,e,f\},\{b,d,f\}\}$

One can remark here that the obtained extensions are often subsets of the original extensions. They can be considered as the core of these extensions, so being more important arguments than the one that are not selected.

Conclusion

Extension-based semantics and ranking based-semantics offer different evaluations of abstract argumentation frameworks. While they can be used separately for different applications, it can be interesting to try to combine these two

approaches, in order to benefit from both kinds of evaluation. This is this path that we initiated in this work. We proposed several ways to combine these approaches and show that each time the obtained semantics have good logical properties.

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