Argumentation Ranking Semantics based on Propagation [1]

Elise BONZON [a], Jérôme DELOBELLE [b], Sébastien KONIECZNY [b] and Nicolas MAUDET [c]

[a] *LIPADE, Université Paris Descartes, Paris, France - bonzon@parisdescartes.fr*
[b] *CRIL, Université d'Artois, Lens, France - {delobelle,konieczny}@cril.fr*
[c] *LIP6, Université Pierre et Marie Curie, Paris, France - nicolas.maudet@lip6.fr*

Abstract. Argumentation is based on the exchange and the evaluation of interacting arguments. Unlike Dung's theory where arguments are either accepted or rejected, ranking-based semantics rank-order arguments from the most to the least acceptable ones. We propose in this work six new ranking-based semantics. We argue that, contrarily to existing ranking semantics in the literature, that focus on evaluating attacks and defenses only, it is reasonable to give a prominent role to non-attacked arguments, as it is the case in standard Dung's semantics. Our six semantics are based on the propagation of the weight of each argument to its neighbors, where the weight of non-attacked arguments is greater than the attacked ones.

Keywords. Argumentation, Ranking-based semantics, Propagation

1. Introduction

Argumentation is a very natural framework for representing conflicting information. A proof of its appeal is the recent development of online platforms where people participate in debates using argumentation graphs (e.g. *debategraph.org* or *arguman.org*) such representation tools become more and more popular.

The question now goes towards the reasoning part: how to automatically use these argumentation graphs that are constructed this way? Argumentation has been a very active topic in Artificial Intelligence since more than two decades now, and Dung's work on abstract argumentation framework [1] can be used to represent the graphs (even if some additional information should be also represented, like the number of people that agree/disagree with an argument and/or an attack, or a support between arguments, . . .). But the main issue is about the semantics that one should use in this case. In fact classical Dung's semantics, using extensions [1] (or equivalently labellings [2]), with their dichotomous evaluation of arguments (accepted/rejected) do not seem very well suited for such applications. As discussed in [3], on such online platforms, with a big number of arguments, and a lot of individuals participating, it can be problematic (in particular quite unintuitive for the participants) to have such a drastic evaluation, that is not that in-

[1] This work benefited from the support of the project AMANDE ANR-13-BS02-0004 of the French National Research Agency (ANR).

formative (since there are only two levels of acceptability), or to propose several possible results (different extensions). So in this case a finer evaluation of arguments seems to be more adequate. The idea is then to have ranking-based semantics, that allow to produce a full ranking of the arguments, from the most to the least acceptable ones. This kind of semantics seems very natural, and it is then quite surprising that they have received little attention until recently [3,4,5,6,7,8,9,10]. These semantics basically rely on the attacks and defenses of each argument in order to evaluate its acceptability rank.

In this work we propose a new family of semantics, that relies on attacks and defenses, like previous semantics, but that also puts a strong emphasis on non-attacked arguments. While many principles remain discussed and controversial, all semantics agree on the fact that non-attacked arguments should have the highest rank. The idea of our semantics is that these arguments should also have a greater impact on the evaluation of the other ones.

In this paper, we propose six new semantics based on the idea of propagation. Each argument has an initial weight that depends on its status (non-attacked arguments have a greater value than attacked ones), and then these weights are progressively propagated to their neighbors. Of course at each propagation the polarity of the weight changes in order to comply with the attack relation meaning. The difference between these semantics lies in the method that is chosen to differentiate non-attacked arguments and attacked ones, and in the choice of considering one or all paths between arguments.

In the next section, we define the notions and notations we will need to define our six propagation semantics in Section 3. Section 4 recalls the logical properties proposed in the literature for ranking-based semantics, and studies which ones are satisfied by our semantics. In Section 5, we study the links between our semantics and previous ones. Section 6 provides an example in order to illustrate the behaviour of the propagation semantics and to relate them to other semantics.

2. Background

Following [1], an argumentation system is a (finite) set of arguments together with the binary conflicts among them.

Definition 1. *An* **argumentation framework (AF)** *is a directed graph* $\langle A, \hookrightarrow \rangle$ *where the set of nodes A is a finite set of* **arguments***, and the set of edges* $\hookrightarrow \subseteq A \times A$ *is an* **attack relation** *between arguments. A set of arguments* $C \subseteq A$ **attacks** *an argument* $b \in A$*, if there exists* $a \in C$*, such that* $(a,b) \in \hookrightarrow$*. C* **defends** *a iff* $\forall b \in A$ *such that* $(b,a) \in \hookrightarrow$*,* $\exists c \in C$ *such that* $(c,b) \in \hookrightarrow$*.*

In the following, $\mathbb{AF}$ will represent the set of all argumentation frameworks.

Definition 2. *Let* $AF = \langle A, \hookrightarrow \rangle$ *and* $a, b \in A$*. A* **path** *from a to b, denoted by* $a \rightsquigarrow b$*, is a sequence of nodes* $s = \langle a_0, \ldots, a_n \rangle$ *such that from each node there is an edge to the next node in the sequence :* $a_0 = a$*,* $a_n = b$ *and* $\forall i < n, (a_i, a_{i+1}) \in \hookrightarrow$*. Its* **length** *is noted* $|a \rightsquigarrow b|$ *and is equal to the number of edges it is composed of.*

In order to encode the fact that there are several possible paths between two arguments, we introduce the notion of multiset of attackers and defenders of an argument.

Definition 3. *Let $AF = \langle A, \hookrightarrow \rangle$ and* a,b $\in A$. *Let $\oplus \in \{M, S\}$, where M (resp. S) stands for multiset (resp. set). The (multi)set of arguments such that there exists a path to* a *with a length of n is denoted by* $\downarrow_n^{\oplus}(\mathrm{a}) = \{\mathrm{b} \mid \exists \mathrm{b} \rightsquigarrow \mathrm{a},$ *with* $|\mathrm{b} \rightsquigarrow \mathrm{a}| = n\}$. *An argument* $\mathrm{b} \in \downarrow_n^{\oplus}(\mathrm{a})$ *is a* **defender** *(resp.* **attacker***) of* a *if $n \in 2\mathbb{N}$ (resp. $n \in 2\mathbb{N}+1$). Let us note* $\downarrow^{\oplus}(\mathrm{a}) = \bigcup_n \downarrow_n^{\oplus}(\mathrm{a})$.

Note that the *direct attackers* of an argument a belong to $\downarrow_1^{\oplus}(a)$.

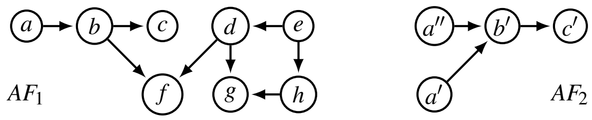

Figure 1. Two argumentation frameworks

Let us discuss how using sets or multisets can influence the result of the ranking-based semantics. Obviously there is no change for the direct attackers because an argument cannot be directly attacked several times by the same argument. However, several paths of length greater than one between two arguments can exist: on AF_1 (Figure 1) there are two paths of length 2 from e to g: $\langle e, d, g \rangle$ and $\langle e, h, g \rangle$. So with sets $\downarrow_2^{S}(g) = \{e\}$, whereas with multisets the result is $\downarrow_2^{M}(g) = \{e, e\}$.

The use of sets can be seen as a focus on the arguments at the end of the path without taking into account the number of possible paths, whereas the multisets encodes the fact that there are several possible paths.

Definition 4. *Let $\oplus \in \{M, S\}$. A path from b to a is a* **branch** *if b is not attacked, that is if $\downarrow_1^{\oplus}(b) = \emptyset$. It is a* **defense branch** *(resp.* **attack branch***) if b is a defender (resp. attacker) of a.*

One of the main goals of argumentation theory is to identify which arguments are rationally *acceptable* according to different notions of acceptability. In [1], the acceptability of an argument depends on its membership to some extensions, whereas ranking-based semantics aim to rank arguments from the most to the least acceptable ones.

Definition 5. *A* **ranking-based semantics** σ *is a function that transforms any argumentation framework $AF = \langle A, \hookrightarrow \rangle$ into a ranking $\succeq_{AF}^{\sigma}$ on A, where $\succeq_{AF}^{\sigma}$ is a preorder (a reflexive and transitive relation) on A. $a \succeq_{AF}^{\sigma} b$ means that a is at least as acceptable as b ($a \simeq_{AF}^{\sigma} b$ is a shortcut for $a \succeq_{AF}^{\sigma} b$ and $b \succeq_{AF}^{\sigma} a$, and $a \succ_{AF}^{\sigma} b$ is a shortcut for $a \succeq_{AF}^{\sigma} b$ and $b \not\succeq_{AF}^{\sigma} a$).*

When there is no ambiguity about the argumentation framework in question, we will use $\succeq^{\sigma}$ instead of $\succeq_{AF}^{\sigma}$.

Finally, we need to introduce the notion of lexicographical order and a shuffle operation between vectors of real number in order to define our new ranking-based semantics.

Definition 6. *A* **lexicographical order** *between two vectors of real numbers $V = \langle V_1, \ldots, V_n \rangle$ and $V' = \langle V'_1, \ldots, V'_n \rangle$ is defined as $V \succ_{lex} V'$ iff $\exists i \leq n$ s.t. $V_i > V'_i$ and $\forall j < i, V_j = V'_j$. $V \simeq_{lex} V'$ means that $V \not\succ_{lex} V'$ and $V' \not\succ_{lex} V$; and $V \succeq_{lex} V'$ means that $V' \not\succ_{lex} V$.*

Definition 7. *The* **shuffle** $\cup_s$ *between two vectors of real numbers $V = \langle V_1, \ldots, V_n \rangle$ and $V' = \langle V'_1, \ldots, V'_n \rangle$ is defined as $V \cup_s V' = \langle V_1, V'_1, V_2, V'_2, \ldots, V_n, V'_n \rangle$.*

3. Propagation Semantics

A standard principle of existing ranking-based semantics is to base the evaluation of an argument on the number of its attackers and of its defenders: the less attackers and the more defenders an argument, the more acceptable the argument. For example, if we compare AF_1 with AF_2 in Figure 1, b is better than b' because b has less attackers than b' (only one for b against two for b'). Inversely, c' has more defenders than c (two for c' against one for c) and the same number of attackers so c' can be considered better than c.

Another important principle to take into account is the role and impact of non-attacked arguments. For example, in AF_1, as a is non-attacked and c is defended by a against the attack of b, a and c are both accepted with respect to Dung's semantics. However, one could go further and say that a is better than c because a is not attacked, whereas c is attacked and is accepted only because of the defense of a. So, we can clearly see that a, and more generally the non-attacked arguments, play a key role in the (classical) acceptability of an argument. Thus, our propagation method will allow those non-attacked arguments to play a key role in the ranking of arguments.

Our approach is based on these two principles. The propagation methods are defined in two steps. The first step consists in assigning a positive initial weight to each argument. The score of 1 attached to non-attacked arguments is set to be higher than the score of attacked arguments, which is an ε between 0 and 1. The value of this ε is chosen accordingly to the degree of influence of the non-attacked arguments that we want: the smaller the value of ε is, the more important the influence of non-attacked arguments on the order prevails. Then, during the second step, we propagate the weights into the graph in changing their polarities in order to comply with the attack relation meaning (attack or defense). For each argument, we accumulate and store the weights from its attackers and defenders in the argumentation framework.

Definition 8. *Let $F = \langle A, \hookrightarrow \rangle$ be an AF. The valuation P of $a \in A$, at step i, is given by:*

$$P_i^{\varepsilon,\oplus}(a) = \begin{cases} v_\varepsilon(a) & if\ i = 0 \\ P_{i-1}^{\varepsilon,\oplus}(a) + (-1)^i \sum\limits_{b \in \downarrow_i^{\oplus}(a)} v_\varepsilon(b) & otherwise \end{cases}$$

where $v : A \to \mathbb{R}^+$ is a valuation function giving an initial weight to each argument, with $\varepsilon \in [0,1]$ such that $\forall b \in A$, $v_\varepsilon(b) = 1$ if $\downarrow_1^{\oplus}(b) = \emptyset$; $v_\varepsilon(b) = \varepsilon$ otherwise.

The **Propagation vector of** *a is denoted $P^{\varepsilon,\oplus}(a) = \langle P_0^{\varepsilon,\oplus}(a), P_1^{\varepsilon,\oplus}(a), \ldots \rangle$.*

Example 1. *Let us calculate the value of P when $\varepsilon = 0.75$ for AF_1 (Figure 1). If no distinction exists between set and multiset then the value is put in the same cell (Table 1). Otherwise, the cell is divided into two parts (valuation for set at left and for multiset at right). For instance, when $i = 2$, $P_2^{0.75,S}(c) = P_2^{0.75,M}(c) = 1$ but $P_2^{0.75,S}(g) = 0.25$ whereas $P_2^{0.75,M}(g) = 1.25$.*

In Table 1, argument f begins with an initial weight of 0.75 ($P_0^{0.75,\oplus}(f) = 0.75$) because it is attacked. Then, during the second step, the direct attackers (b and d which are also attacked) propagate negatively their weights of 0.75 to f, so $P_1^{0.75,\oplus}(f) = P_0^{0.75,\oplus}(f) - (v_{0.75}(d) + v_{0.75}(b)) = -0.75$. Finally, during the third step, f receives

$P_i^{0.75,\oplus}$	a,e		b,d,h		c		f		g	
	S	M	S	M	S	M	S	M	S	M
0	1		0.75		0.75		0.75		0.75	
1	1		-0.25		0		-0.75		-0.75	
2	1		-0.25		1		1.25		0.25	1.25

Table 1. Computation of propagation vector when $\varepsilon = 0.75$

positively the weights of 1 from a and e which are non-attacked, so $P_2^{0.75,\oplus}(f) = P_1^{0.75,\oplus}(f) + (v_{0.75}(a) + v_{0.75}(e)) = 1.25$. *As there exists no path to f with a length higher than 2, this score remains the same, and* $P^{0.75,\oplus}(f) = \langle 0.75, -0.75, 1.25, 1.25, \dots \rangle$.

It is important to note that $P^{\varepsilon,\oplus}(a)$ may be infinite (this may occur when an argument is involved in at least one cycle). Moreover, the valuation $P_n^{\varepsilon,\oplus}(x)$ of an argument x is not even necessarily bounded as $n \to \infty$. After a finite number of steps though, an argument is bound to receive the influence of exactly the same arguments than in a previous step of the vector (which means that the vector can be finitely encoded). More precisely, this number of steps is in the order of the least common multiplier of the cycle lengths occurring in the argumentation graph. As a ranking-based semantics is not concerned with the exact values of arguments, but only in their relative ordering, this is sufficient for our purpose.

3.1. *Propa*$_\varepsilon$

Once the propagation vector is calculated for each argument in the argumentation framework, we can compare the different vectors in order to obtain an order between all the arguments. We want the influence of arguments to quickly decrease with the length of a path, so an option is to use a lexicographical comparison for comparing these vectors. For the first semantics we just compare the propagation vectors for a given ε.

Definition 9. *Let* $\oplus \in \{M, S\}$. *The ranking-based semantics Propa*$_\varepsilon^{\varepsilon,\oplus}$ *associates to any AF =* $\langle A, \hookrightarrow \rangle$ *a ranking* $\succeq_{AF}^{P_\oplus}$ *on A such that* $\forall a, b \in A$, $a \succeq_{AF}^{P_\oplus} b$ *iff* $P^{\varepsilon,\oplus}(a) \succeq_{lex} P^{\varepsilon,\oplus}(b)$.

So this defines two semantics, one using sets and one using multisets for the attack and defense branches computations.

Example 1 (cont.). *In Table 1, if* $\oplus = S$, *we obtain the ranking* $a \simeq^{P_S} e \succ^{P_S} c \succ^{P_S} b \simeq^{P_S} d \simeq^{P_S} h \succ^{P_S} f \succ^{P_S} g$. *If* $\oplus = M$, *we have* $a \simeq^{P_M} e \succ^{P_M} c \succ^{P_M} b \simeq^{P_M} d \simeq^{P_M} h \succ^{P_M} f \simeq^{P_M} g$.

These semantics focus mainly on the attackers and defenders in adding the fact that if there exists non-attacked arguments among them, these ones will be more influential than attacked arguments. But this influence depends also on the value of ε. Indeed, two values of ε can lead to different orders. On Example 1, with $\varepsilon = 0.75$, if we focus on f, which is defended twice, and h, which is attacked (and not defended), we can see that h is better than f because $P_1^{0.75,\oplus}(f) < P_1^{0.75,\oplus}(h)$. But if we take $\varepsilon < 0.5$, we obtain the opposite case. For example, with $\varepsilon = 0.3$, we have $P^{0.3,\oplus}(f) = \langle 0.3, -0.3, 1.7, \dots \rangle$ and $P^{0.3,\oplus}(h) = \langle 0.3, -0.7, \dots \rangle$. With the lexicographical order, f is now better than h because $P_1^{0.3,\oplus}(f) > P_1^{0.3,\oplus}(h)$.

So, with Propa$_\varepsilon$ semantics, an argument with only (but numerous) defense branches can be worse than an argument only attacked by one non-attacked argument. It is a possible point of view to focus only on the attackers in saying that the more an argument is directly attacked, the less acceptable the argument. It is the case, for instance, with the semantics proposed by Amgoud and Ben-Naim [8]. But other approaches are possible, as we shall see now.

3.2. *Propa*$_{1+\varepsilon}$

If we do not want the influence of non-attacked arguments to be drown in by the influence of attacked arguments, we have to split and lexicographically compare the influence of the two kinds of arguments.

Definition 10. *Let $\oplus \in \{M,S\}$. The ranking-based semantics $Propa^{\varepsilon,\oplus}_{1+\varepsilon}$ associates to any AF = $\langle A, \hookrightarrow\rangle$ a ranking $\succeq^{\widehat{P}_\oplus}_{AF}$ on A such that $\forall a,b \in A$,*

$$a \succeq^{\widehat{P}_\oplus}_{AF} b \text{ iff } P^{0,\oplus}(a) \cup_s P^{\varepsilon,\oplus}(a) \succeq_{lex} P^{0,\oplus}(b) \cup_s P^{\varepsilon,\oplus}(b)$$

With these semantics, we simultaneously look at the result of the two propagation vectors $P^{0,\oplus}$ and $P^{\varepsilon,\oplus}$ step by step, using the shuffle operation, starting with the first value of the propagation vector $P^{0,\oplus}$ (i.e. the one that takes into account non-attacked arguments only). In the case where two arguments are still equally acceptable, we compare the first value of the propagation vector $P^{\varepsilon,\oplus}$. Then, in case of equality, we move to the second step and so on.

Example 1 (cont.). *Let us focus on f with the two propagation vectors: $P^{0,\oplus}(f) = \langle 0,0,2,\dots\rangle$ (see Table 2 where $\varepsilon = 0$) and $P^{0.75,\oplus}(f) = \langle 0.75,-0.75,1.25,\dots\rangle$ (see Table 1 where $\varepsilon = 0.75$). We use the shuffle $\cup_s$ to combine the previous propagation vectors: $P^{0,\oplus}(f) \cup_s P^{0.75,\oplus}(f) = \langle 0,0.75,0,-0.75,2,1.25,\dots\rangle$. We apply the same method for all the others arguments and we use the lexicographical order to compare them. So if $\oplus = S$, we obtain the ranking $a \simeq^{\widehat{P}_S} e \succ^{\widehat{P}_S} c \succ^{\widehat{P}_S} f \succ^{\widehat{P}_S} g \succ^{\widehat{P}_S} b \simeq^{\widehat{P}_S} d \simeq^{\widehat{P}_S} h$ whereas if $\oplus = M$, we have $a \simeq^{\widehat{P}_M} e \succ^{\widehat{P}_M} c \succ^{\widehat{P}_M} f \simeq^{\widehat{P}_M} g \succ^{\widehat{P}_M} b \simeq^{\widehat{P}_M} d \simeq^{\widehat{P}_M} h$.*

<table>
<tr><th rowspan="2">$P^{0,\oplus}_i$</th><th colspan="2">a,e</th><th colspan="2">b,d,h</th><th colspan="2">c</th><th colspan="2">f</th><th colspan="2">g</th></tr>
<tr><th>S</th><th>M</th><th>S</th><th>M</th><th>S</th><th>M</th><th>S</th><th>M</th><th>S</th><th>M</th></tr>
<tr><td>0</td><td colspan="2">1</td><td colspan="2">0</td><td colspan="2">0</td><td colspan="2">0</td><td colspan="2">0</td></tr>
<tr><td>1</td><td colspan="2">1</td><td colspan="2">-1</td><td colspan="2">0</td><td colspan="2">0</td><td colspan="2">0</td></tr>
<tr><td>2</td><td colspan="2">1</td><td colspan="2">-1</td><td colspan="2">1</td><td colspan="2">2</td><td>1</td><td>2</td></tr>
</table>

Table 2. Computation of propagation vector when $\varepsilon = 0$

It is also important to notice that $Propa_{1+\varepsilon}$, conversely to $Propa_\varepsilon$, returns the same order whatever the value of ε, that removes the problem of choosing "a good" ε:

Proposition 1. *Let $\oplus \in \{M,S\}$, $\forall AF \in \mathbb{AF}$ and $\forall \varepsilon, \varepsilon' \in\,]0,1]$,*

$$Propa^{\varepsilon,\oplus}_{1+\varepsilon}(AF) = Propa^{\varepsilon',\oplus}_{1+\varepsilon}(AF)$$

3.3. *Propa*$_{1\to\varepsilon}$

A last possibility is to give a higher priority to the non-attacked arguments, by propagating only their weights in the graph. And if two arguments are equivalent for this comparison, they are compared using the $Propa_\varepsilon$ method. Technically, the priority to the non-attacked arguments is given by using $\varepsilon = 0$. So we compare first the propagation vector $P^{0,\oplus}$. And if the two propagation vectors are identical, we restart with a non-zero ε:

Definition 11. *Let $\oplus \in \{M,S\}$. The ranking-based semantics $Propa^{\varepsilon,\oplus}_{1\to\varepsilon}$ associates to any AF = $\langle A, \hookrightarrow\rangle$ a ranking $\succeq^{\overline{P}_{\oplus}}_{AF}$ on A such that $\forall a,b \in A$,*

$$a \succeq^{\overline{P}_{\oplus}}_{AF} b \text{ iff } P^{0,\oplus}(a) \succ_{lex} P^{0,\oplus}(b) \text{ or, } (P^{0,\oplus}(a) \simeq_{lex} P^{0,\oplus}(b) \text{ and } P^{\varepsilon,\oplus}(a) \succeq_{lex} P^{\varepsilon,\oplus}(b))$$

Example 1 (cont.)**.** *$Propa_{1\to\varepsilon}$ focuses first in Table 2, where $\varepsilon = 0$, and then in Table 1, where $\varepsilon = 0.75$. If $\oplus = S$, we obtain the ranking $a \simeq^{\overline{P}_S} e \succ^{\overline{P}_S} f \succ^{\overline{P}_S} c \succ^{\overline{P}_S} g \succ^{\overline{P}_S} b \simeq^{\overline{P}_S} d \simeq^{\overline{P}_S} h$ whereas if $\oplus = M$, we have $a \simeq^{\overline{P}_M} e \succ^{\overline{P}_M} f \simeq^{\overline{P}_M} g \succ^{\overline{P}_M} c \succ^{\overline{P}_M} b \simeq^{\overline{P}_M} d \simeq^{\overline{P}_M} h$.*

With these semantics, an argument with a lot of defense branches will receive a lot of positive weights, and conversely, an argument with a lot of attack branches, will receive a lot of negative weights. Thus, as f and g have one more defense branch than c (with the multiset), which has also one more defense branch than b, d and h, we have that f and g are better than c which is better than b, d and h. However, focusing only on $\varepsilon = 0$, we cannot distinguish the arguments with the same number of defense/attack branches. This is why we use the propagation vector with $\varepsilon \neq 0$ to decide between those.

It is also important to notice that $Propa_{1\to\varepsilon}$, like $Propa_{1+\varepsilon}$, returns the same order whatever the value of ε :

Proposition 2. *Let $\oplus \in \{M,S\}$, $\forall AF \in \mathbb{AF}$ and $\forall \varepsilon, \varepsilon' \in]0,1]$,*

$$Propa^{\varepsilon,\oplus}_{1\to\varepsilon}(AF) = Propa^{\varepsilon',\oplus}_{1\to\varepsilon}(AF)$$

4. Properties

4.1. Properties for ranking-based semantics

In the last few years, a set of properties have been proposed in different papers, allowing to better understand the behavior of the different ranking-based semantics. We adopt the recent catalogue of properties listed in [11] (for space reasons we point out to this paper for their formal definitions).

One can find the properties Abs, In, VP, DP, CT, SCT, CP, QP and DDP in [8], the properties In, VP and SC in [6], the properties $\oplus$DB, +DB, $\uparrow$AB, +AB, $\uparrow$DB in [5,11], and the properties Tot and AvsFD in [11]. We do not claim that all these properties are mandatory (in particular some of them are incompatible and we do not necessarily endorse all of them). Let a and b two arguments in an AF.

Abstraction (Abs) The arguments' ranking should only depend on the attack relation.

Independence (In) The ranking between two arguments should be independent of arguments that are not connected to either of them.

Void Precedence (VP) A non-attacked argument should be strictly more acceptable than an attacked argument.

Self-Contradiction (SC) An argument that attacks itself should be strictly less acceptable than an argument that does not.

Cardinality Precedence (CP) If a has strictly more direct attackers than b, then b should be strictly more acceptable than a.

Quality Precedence (QP) If a has a direct attacker strictly more acceptable than any direct attacker of b, then a should be strictly more acceptable than b.

Counter-Transitivity (CT) If the direct attackers of b are at least as numerous and acceptable as those of a, then a should be at least as acceptable as b.
Strict Counter-Transitivity (SCT) If CT is satisfied and if the direct attackers of b are either strictly more numerous, or strictly more acceptable than those of a, then a should be strictly more acceptable than b.
Defense Precedence (DP) If arguments a and b have the same number of direct attackers, and if a is defended at least once whereas b is not, a should be ranked higher than b.
Distributed-Defense Precedence (DDP) A defense where each defender attacks a distinct attacker is better than any other.
In order to introduce the next properties, let us define the notion of ancestor's graph:

Definition 12. *Let $AF = \langle A, \hookrightarrow \rangle$ and $a \in A$. The* ***ancestor's graph*** *of a is denoted by $Anc(a) = \langle A', \hookrightarrow' \rangle$ with $A' = \downarrow^S(a)$ and $\hookrightarrow' = \{(a_1, a_2) \in \hookrightarrow \mid a_1 \in A'$ and $a_2 \in A'\}$.*

Strict addition of Defense Branch (⊕DB) If a and b have the same ancestor's graph, except that a has an additional defense branch, then a should be strictly more acceptable than b.
Addition of Defense Branch (+DB) If a and b have the same ancestor's graph, which is not empty, except than a has an additional defense branch, then a should be strictly more acceptable than b.
Increase of Attack Branch (↑AB) If a and b have the same ancestor's graph, except than one attack branch of a is longer than for b, then a should be strictly more acceptable than b.
Addition of Attack Branch (+AB) If a and b have the same ancestor's graph, except that a has an additional attack branch, then a should be strictly less acceptable than b.
Increase of Defense Branch (↑DB) If a and b have the same ancestor's graph, except than one defense branch of a is longer than for b, then a should be strictly less acceptable than b.
Total (Tot) All arguments can be compared.
Attack vs Full Defense (AvsFD) A fully defended argument (without any attack branch) should be strictly more acceptable than an argument attacked once by a non-attacked argument.

Finally, we introduce a new property, which generalizes the property Non-attacked Equivalence [11], that says that all the non-attacked arguments are equally acceptable.
Argument Equivalence (AE) If two arguments have the same ancestor's graph, then they are equally acceptable.
It is important to note that the reverse is not always true. Indeed, on Example 1, f and g have different ancestor's graphs, *but* are equally acceptable when taking the multiset.

4.2. Properties Satisfied by Propagation

We are now in a position to check which of these properties are satisfied by our six ranking-based semantics based on propagation. A first remark is that if we choose $\varepsilon = 0$, the three kinds of semantics return exactly the same order.

Proposition 3. *Let $\oplus \in \{M, S\}$, for all $AF \in \mathbb{AF}$,*

$$Propa_{\varepsilon}^{0,\oplus}(AF) = Propa_{1+\varepsilon}^{0,\oplus}(AF) = Propa_{1\to\varepsilon}^{0,\oplus}(AF)$$

In this case, only the weights propagated by the non-attacked arguments are taken into account. This can make sense, but when there is no non-attacked argument in the AF, all the arguments have a propagation vector composed only of 0, therefore trivializing the result. So we remove this possibility for studying the properties.

Let us begin to check which properties are satisfied by $Propa_\varepsilon$

Proposition 4. *Let $\oplus \in \{M,S\}$ and $\varepsilon \in]0,1]$. $Propa_\varepsilon^{\varepsilon,\oplus}$ satisfies Abs, In, VP, DP, CT, SCT, ↑AB, ↑DB, +AB, Tot and AE. The other properties are not satisfied.*

$Propa_{1+\varepsilon}$ satisfies more properties.

Proposition 5. *Let $\oplus \in \{M,S\}$ and $\varepsilon \in]0,1]$. $Propa_{1+\varepsilon}^{\varepsilon,\oplus}$ satisfies Abs, In, VP, DP, CT, SCT, DDP, ↑AB, ↑DB, +AB, Tot, AE and AvsFD. The other properties are not satisfied.*

Finally, let us list the properties satisfied by $Propa_{1\to\varepsilon}$.

Proposition 6. *Let $\oplus \in \{M,S\}$ and $\varepsilon \in]0,1]$. $Propa_{1\to\varepsilon}^{\varepsilon,\oplus}$ satisfies Abs, In, VP, DP, DDP, +DB, ↑AB, ↑DB, +AB, Tot, AE and AvsFD. The other properties are not satisfied.*

First it is interesting to remark than choosing sets or multisets for the definitions, although clearly leading to different semantics, do not have any impact on the satisfaction of these properties. This may suggest that some properties allowing to make such a distinction are still missing.

Without surprise, the semantics satisfy the properties Abs, In, VP, DP, +AB and Tot, which are expected according to [11]. Contrastingly, CP, QP, SC and ⊕DB are not satisfied by our semantics, but they describe very specific ranking-based semantics behaviors, which differ from the ones designed here. First of all, ⊕DB is incompatible with VP [11] which is satisfied by all our semantics. CP and QP focus only on the direct attackers whereas our semantics look also at the impact of the non-attacked arguments in the entire graph. Finally, concerning the property SC, our semantics consider that an argument that attacks itself is a cycle with a length equal to 1. So an argument which attacks itself remains better than an argument which is attacked by a non-attacked argument.

Now, comparing the three families, $Propa_\varepsilon$, $Propa_{1+\varepsilon}$, and $Propa_{1\to\varepsilon}$; $Propa_\varepsilon$ is the only one that does not satisfy DDP and AvsFD. Finally, we can see that $Propa_{1\to\varepsilon}$ is the only one to satisfy +DB and to not satisfy CT and SCT because it is the only one to consider the defense as a reinforcement for the defended argument.

5. Links between Semantics

In this section, we establish the links between the six ranking based-semantics based on propagation, but also between these semantics and some ranking-based semantics existing in the literature. A first important remark is that $Propa_{1+\varepsilon}$ can be seen as a special case of $Propa_\varepsilon$. Let us make this link more formal.

Let $AF = \langle A, \hookrightarrow \rangle \in \mathbb{AF}$ be an argumentation framework, and $\oplus \in \{M,S\}$. We define $maxdeg(AF)$, the maximal indegree of AF, as $maxdeg(AF) = \max_{a\in A} |\downarrow_1^{\oplus}(a)|$.

Proposition 7. *Let $AF = \langle A, \hookrightarrow \rangle \in \mathbb{AF}$. For any $\varepsilon < \frac{1}{maxdeg(AF)}$,*

$$Propa_{1+\varepsilon}^{\varepsilon,\oplus}(AF) = Propa_\varepsilon^{\varepsilon,\oplus}(AF)$$

But as we saw previously with the satisfied properties, even if, in the light of the above result, $Propa_{1+\varepsilon}$ could be considered as particular case of $Propa_\varepsilon$ for a given ε, it forms a sufficiently interesting subclass for being defined and studied on its own right.

In addition to the case where $\varepsilon = 0$, there is another particular situation where all the propagation semantics return the same order: when there exists no non-attacked argument in the argumentation framework.

Proposition 8. *Let* $\oplus \in \{M, S\}$, $\forall AF = \langle A, \hookrightarrow\rangle \in \mathbb{AF}$, $\forall \varepsilon \in]0,1]$, *if* $\nexists a \in A$ *s.t.* $\downarrow_1^{\oplus}(a) = \emptyset$ *then* $Propa_{\varepsilon}^{\varepsilon,\oplus}(AF) = Propa_{1+\varepsilon}^{\varepsilon,\oplus}(AF) = Propa_{1\to\varepsilon}^{\varepsilon,\oplus}(AF)$.

Indeed, if there is no non-attacked argument, for $Propa_{1+\varepsilon}$ and $Propa_{1\to\varepsilon}$, the first case where $\varepsilon = 0$ returns the same propagation vector for all the arguments (for all argument a, $P^{0,\oplus}(a) = \langle 0, 0, \dots\rangle$). Consequently the only way to make a difference between arguments is to look at the case where $\varepsilon \neq 0$ exactly like $Propa_\varepsilon$. In other words, when there is no non-attacked argument, the semantics compare the arguments only on the number of attackers/defenders.

In this case, a link can be established between our ranking-based semantics and one semantics of the literature. The Discussion-based semantics [8] compares arguments by counting the number of direct attackers. If this number is the same for some arguments, the size of paths is recursively increased until a difference is found:

Definition 13. **[8]** *Let* $AF = \langle A, \hookrightarrow\rangle$, $a \in A$, *and* $i \in \mathbb{N}$. *Let* $Dis_i(a) = (-1)^{i+1}|\downarrow_i^M(a)|$, *and* $Dis(a) = \langle Dis_1(a), Dis_2(a), \dots\rangle$. *The* **ranking-based semantics Dbs** *associates to* AF *a ranking* $\succeq_{AF}^{Dbs}$ *on* A *such that* $\forall a, b \in A$, $a \succeq_{AF}^{Dbs} b$ *iff* $Dis(b) \succeq_{lex} Dis(a)$.

Dbs and the propagation semantics share similar principles regarding the way paths are counting and use the lexicographical comparison. However, let us recall that, in the general case, our semantics also take into account the role of the non-attacked arguments which has consequences on the order between arguments. But in the case where there is no non-attacked argument, the order returned by both semantics is the same.

Proposition 9. $\forall AF = \langle A, \hookrightarrow\rangle \in \mathbb{AF}$, $\forall \varepsilon \in]0,1]$, *if* $\nexists a \in A$ *such that* $\downarrow_1^M(a) = \emptyset$, *then* $Propa_{\varepsilon}^{\varepsilon,M}(AF) = Propa_{1+\varepsilon}^{\varepsilon,M}(AF) = Propa_{1\to\varepsilon}^{\varepsilon,M}(AF) = Dbs(AF)$.

Note that this result is obtained with the multiset version of the three kinds of semantics. The set versions are not equivalent.

6. Example

In this section, we apply the different existing ranking-based semantics and the six semantics based on propagation on an example with few arguments. The objective is to illustrate their behaviors with regard to some particular situations. We consider the semantics based on Social Argumentation Frameworks *SAF* [3], the semantics Categoriser *Cat* [4,9], the semantics based on tuple[2] values $Tuples^*$ [5], the semantics proposed by Matt and Toni *M&T* [6], the semantics proposed by Grossi and Modgil *G&M* [10], the Discussion-based semantics *Dbs* and the Burden-based semantics *Bbs* [8].

[2] In order to avoid infinite tuples, we consider this approach for acyclic graph only. See [11] for details.

Example 1 (cont.). *For a better visibility of the obtained orders, we do not consider the argument e which is similar to a (both are non-attacked) and the arguments d and h which are similar to b (all attacked once by one non-attacked argument) in the final pre-order because they are always equally acceptable.*

Semantics	*Order between arguments*
Cat	$a \succ^{\text{Cat}} c \succ^{\text{Cat}} b \simeq^{\text{Cat}} f \simeq^{\text{Cat}} g$
SAF	$a \succ^{\text{SAF}} c \succ^{\text{SAF}} f \simeq^{\text{SAF}} g \succ^{\text{SAF}} b$
Tuples*	$a \succ^{\text{T}} f \simeq^{\text{T}} g \succ^{\text{T}} c \succ^{\text{T}} b$
M&T	$a \succ^{\text{MT}} c \simeq^{\text{MT}} f \simeq^{\text{MT}} g \succ^{\text{MT}} b$
G&M	$a \succ^{\text{GM}} c \succ^{\text{GM}} f \simeq^{\text{GM}} g \succ^{\text{GM}} b$
Dbs	$a \succ^{\text{Dbs}} c \succ^{\text{Dbs}} b \succ^{\text{Dbs}} f \simeq^{\text{Dbs}} g$
Bbs	$a \succ^{\text{Bbs}} c \succ^{\text{Bbs}} b \succ^{\text{Bbs}} f \simeq^{\text{Bbs}} g$

$\text{Propa}_{\varepsilon}^{0.75,S}$	$a \succ^{P_S} c \succ^{P_S} b \succ^{P_S} f \succ^{P_S} g$
$\text{Propa}_{\varepsilon}^{0.3,S}$	$a \succ^{P_S} c \succ^{P_S} f \succ^{P_S} g \succ^{P_S} b$
$\text{Propa}_{\varepsilon}^{0.75,M}$	$a \succ^{P_M} c \succ^{P_M} b \succ^{P_M} f \simeq^{P_M} g$
$\text{Propa}_{\varepsilon}^{0.3,M}$	$a \succ^{P_M} c \succ^{P_M} f \simeq^{P_M} g \succ^{P_M} b$
$\text{Propa}_{1\to\varepsilon}^{\varepsilon,S}$	$a \succ^{\overline{P}_S} f \succ^{\overline{P}_S} c \succ^{\overline{P}_S} g \succ^{\overline{P}_S} b$
$\text{Propa}_{1\to\varepsilon}^{\varepsilon,M}$	$a \succ^{\overline{P}_M} f \simeq^{\overline{P}_M} g \succ^{\overline{P}_M} c \succ^{\overline{P}_M} b$
$\text{Propa}_{1+\varepsilon}^{\varepsilon,S}$	$a \succ^{\widehat{P}_S} c \succ^{\widehat{P}_S} f \succ^{\widehat{P}_S} g \succ^{\widehat{P}_S} b$
$\text{Propa}_{1+\varepsilon}^{\varepsilon,M}$	$a \succ^{\widehat{P}_M} c \succ^{\widehat{P}_M} f \simeq^{\widehat{P}_M} g \succ^{\widehat{P}_M} b$

Table 3. Order obtained with the different semantics on AF_1 (Figure 1).

First of all, all semantics consider a (and e) as the best argument because it is non-attacked (see property VP). On the contrary, b (but also d and h) is most of the time the worst argument because it is attacked by the better argument. It is not the case with Dbs and Bbs because they satisfy the property Cardinality Precedence, where the greater the number of direct attackers for an argument, the weaker the level of acceptability of this argument. It is why, for these two semantics, b is better than f and g which are both defended. Note that it is also the case with $Propa_\varepsilon$ *when* $\varepsilon > 0.5$.

It is interesting to note that almost all semantics make no distinction between f and g, both defended twice (by non-attacked arguments). Only our Propagation semantics using sets make a distinction between the two, preferring f that is defended by two arguments, whereas g is defended twice but by the same argument e.

Finally, concerning the three defended arguments (c, f and g), the order reflects the position of the semantics about the notion of defense. We can see that, for $Propa_{1\to\varepsilon}$ *and* $Tuples^*$, *f and g are better than c because they consider a defense as a reinforcement, contrary to all the others semantics.*

7. Conclusion

In this work we proposed six new ranking-based semantics based on the propagation of the weights of arguments, that give a higher weight to non-attacked arguments. The differences between the six semantics lie in the choice of the interaction between attacked and non-attacked arguments (i.e. how much priority do we give to non-attacked arguments), and in the choice of sets or multi-sets as tracking of attacking and defending arguments.

The basic motivating idea behind these semantics, and one of the main contributions of this work, is that one can not take into account only information on attacks and defenses of an argument, but also has to take into account the impact of non-attacked arguments. This idea follows the principle of classical Dung's semantics. However it should be noted that full compatibility of rankings with extensions (sets of mutually acceptable arguments) is a difficult to reach objective, as these semantics do not capture the interaction between arguments and remain at the level of the acceptability of single arguments. For instance, two arguments may be highly ranked but mutually incompatible: ranking-based semantics are blind to this.

We show that these semantics have interesting properties. In particular they satisfy the properties that should be satisfied by any ranking semantics according to [11]. In particular the semantics $Propa_{1+\varepsilon}$ and $Propa_{1\rightarrow\varepsilon}$ satisfy the very natural AvsFD property, that is not satisfied by most of previously proposed ranked-based semantics.

We also show some relationships between these semantics and other ones: all the propagation semantics based on multisets coincide with the semantics Dbs when there is no non-attacked arguments in the AF. So they can be viewed as improvement of Dbs allowing to take into account the impact of non-attacked arguments.

Also, by many respect semantics $Propa_{1\rightarrow\varepsilon}$ is close to the Tuples* semantics [5]. The Tuples* semantics does not necessarily provide a total pre-order, and it cannot be applied (easily) if there is a cycle in the AF. So in a sense $Propa_{1\rightarrow\varepsilon}$ could be seen as an improvement of the ideas of Tuples* that allows to overcome these limitations.

This work on ranked-based semantics is motivated by applications for online debates platforms. On these platforms people can usually vote on arguments and/or on attacks. So this provides weights on the arguments and on the attacks. The SAF framework [3] allows to take these information into account. We started with the basic framework, without any weights. Now the plan is to study the full framework, with weights on attacks and on arguments. We want to study how to generalize these semantics with weights, and to study which are the adaptations of the properties, or the missing ones, in this case.

References

[1] P. M. Dung, "On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games," *Artificial Intelligence*, vol. 77, no. 2, pp. 321–358, 1995.

[2] M. Caminada, "On the issue of reinstatement in argumentation," in *Proc. of the 10th European Conference on Logics in Artificial Intelligence, (JELIA'06)*, pp. 111–123, 2006.

[3] J. Leite and J. Martins, "Social abstract argumentation," in *Proc. of the 22nd International Joint Conference on Artificial Intelligence, (IJCAI'11)*, pp. 2287–2292, 2011.

[4] P. Besnard and A. Hunter, "A logic-based theory of deductive arguments," *Artificial Intelligence*, vol. 128, no. 1-2, pp. 203–235, 2001.

[5] C. Cayrol and M. Lagasquie-Schiex, "Graduality in argumentation," *Journal of Artificial Intelligence Research*, vol. 23, pp. 245–297, 2005.

[6] P. Matt and F. Toni, "A game-theoretic measure of argument strength for abstract argumentation," in *Proc. of the 11th European Conference on Logics in Artificial Intelligence, (JELIA'08)*, pp. 285–297, 2008.

[7] C. da Costa Pereira, A. Tettamanzi, and S. Villata, "Changing one's mind: Erase or rewind?," in *Proc. of the 22nd International Joint Conference on Artificial Intelligence, (IJCAI'11)*, pp. 164–171, 2011.

[8] L. Amgoud and J. Ben-Naim, "Ranking-based semantics for argumentation frameworks," in *Proc. of the 7th International Conference on Scalable Uncertainty Management, (SUM'13)*, pp. 134–147, 2013.

[9] F. Pu, J. Luo, Y. Zhang, and G. Luo, "Argument ranking with categoriser function," in *Proc. of the 7th International Conference on Knowledge Science, Engineering and Management, (KSEM'14)*, pp. 290–301, 2014.

[10] D. Grossi and S. Modgil, "On the graded acceptability of arguments," in *Proc. of the Twenty-Fourth International Joint Conference on Artificial Intelligence, (IJCAI'15)*, pp. 868–874, 2015.

[11] E. Bonzon, J. Delobelle, S. Konieczny, and N. Maudet, "A Comparative Study of Ranking-based Semantics for Abstract Argumentation," in *Proc. of the 30th AAAI Conference on Artificial Intelligence (AAAI'16)*, pp. 914–920, 2016.