Burnt Pancake Problem : New Results

(New Lower Bounds on the Diameter and New Experimental Optimality Ratios)

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Outline

The Burnt Pancake Problem ; Known hard positions : $-I_n$, J_n, Y_n

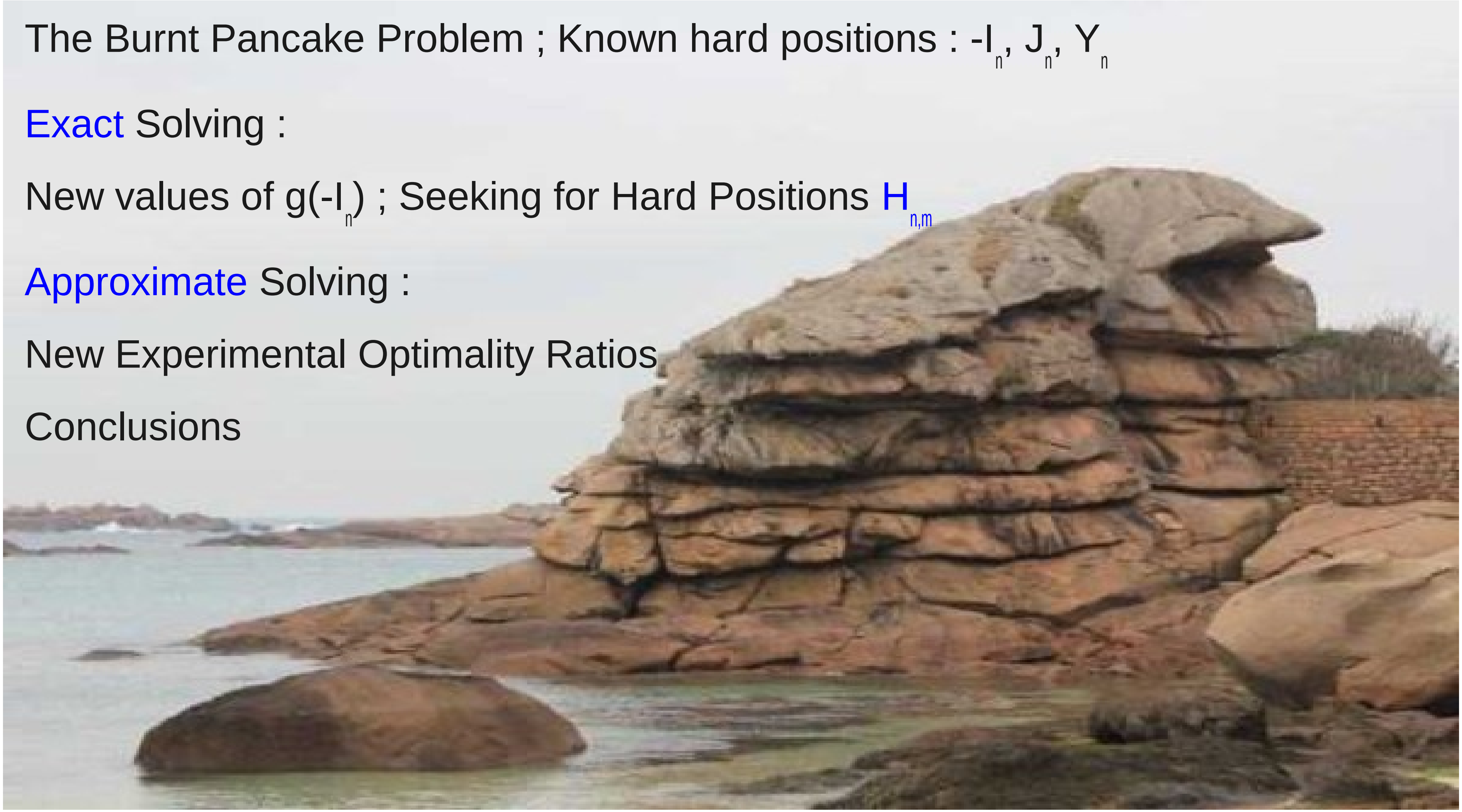
Exact Solving :

New values of $g(-I_n)$; Seeking for Hard Positions $H_{n,m}$

Approximate Solving :

New Experimental Optimality Ratios

Conclusions

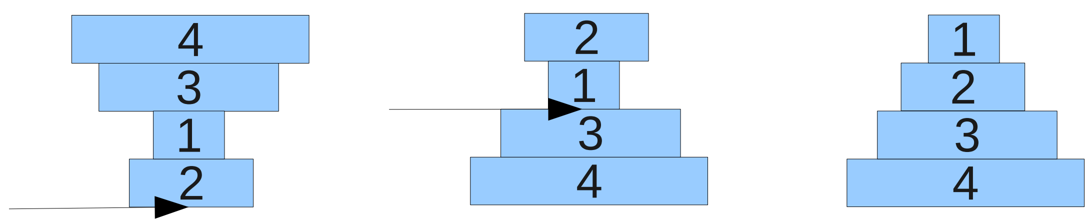


Pancake Problem : example

A stack of pancakes. Each pancake has a size.

Goal : sort the stack with the greatest pancake at the bottom.

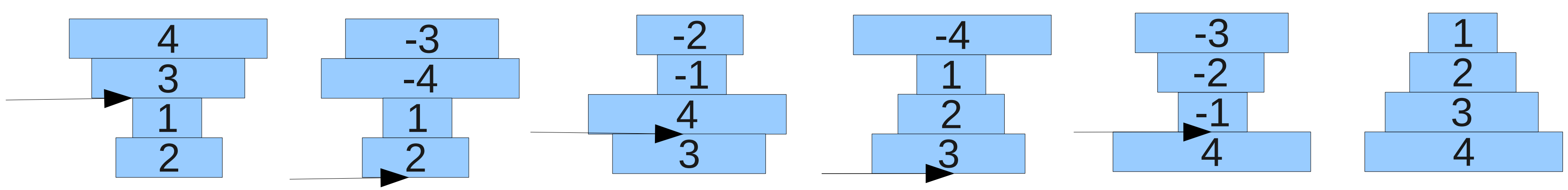
Unburnt case :



L == 2

Burnt case : Each pancake has a burnt side.

L == 5



Goal : the burnt sides have to be oriented downward

Burnt == signed

4 linked classes of problems

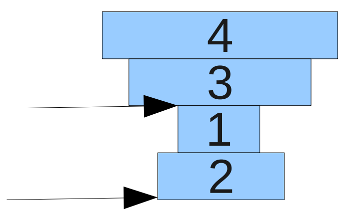
#spatulas signed	1	2
yes	Burnt Pancake ? (Cohen & Blum 1995) (Cibulka 2011)	Signed Genome : Polynomial (Pevzner & Hennenhalli 1995) Linear * (Bader & al 2001) Quadratic (Bergeron 2005)
no	Unburnt Pancake NP-hard (Bulteau & al 2012)	Unsigned Genome NP-hard (Caprara 1997)

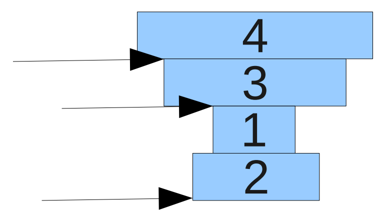
Breakpoint (bp), anti-adjacency (aa)

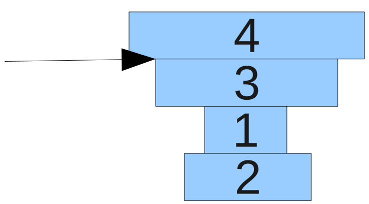
Unburnt | Burnt

Breakpoints | Breakpoints | Anti-adjacencies

Size difference != +-1 | Size difference != 1 | Size difference == -1







#bp == 2 | #bp == 3 | #aa = 1

#bp : number of breakpoints

admissible heuristic (Gates Papadimitriou 1979 ; Helmert 2010)

#aa : number of anti-adjacencies

interesting feature (Cibulka 2011)

$-I_n$

Take the identity stack I_n and reverse the pancakes one by one :

I_4 $-I_4$

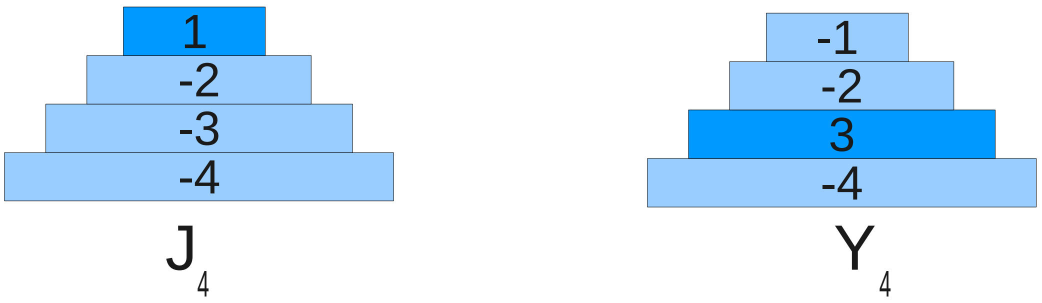
$g(-I_n)$ was known for n <= 20

n	2	3	4	5	...	17	18	19	20	21	22	23	24	25	26	27
$g(-I_n)$	4	6	8	10		28	29	30	32	33	35	36	38	39	41	42

How ? IDA* + #bp

J_n and Y_n

Reverse all the pancakes one by one, except one pancake :



J_n : reverse all the pancakes except the topmost pancake

Y_n : reverse all the pancakes except the second pancake

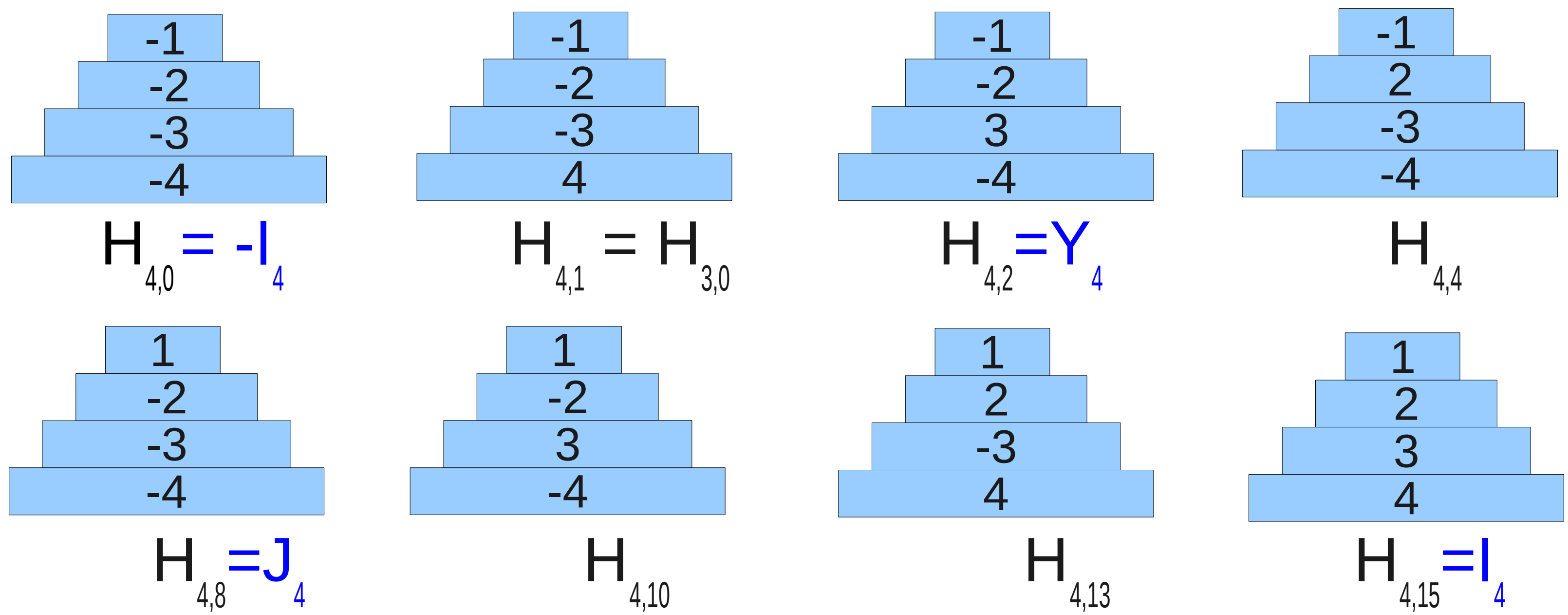
Cohen and Blum conjecture : $-I_n$ is « maximal » for any n

The conjecture is false (Cibulka 2011)

For n=15 : J_{15} and Y_{15} are « maximal » but not $-I_{15}$

$H_{n,m}$

Reverse some pancakes one by one :



m : non negative n bit integer

Each pancake corresponds to one bit of m

The topmost pancake corresponds to the most significant bit of m

$H_{n,m}$: Starting with -In, reverse the pancakes corresponding to bit==1

Seeking for hard positions (1/4)

- B_n : set of hard stacks of size n
- Our light and uncomplete process :

$B_2 = \{ -I_2 \}$

For n=2 to ∞ do

    For any b in B_n do

        For r=1,2,n

            new = reverse r^{th} pancake (b)

            new = add pancake -(n+1) at the bottom

            solve (new)

            Update B_{n+1}

Seeking for hard positions (2/4)

N	d_N	T	B_N
2	4	0s	$-I_2$
3	6	0s	$-I_3$ J_3
4	8	0s	$-I_4$ J_4
5	10	0s	$-I_5$ J_5
6	12	0s	$-I_6$
7	14	0s	$-I_7$
8	15	1s	$-I_8$
9	17	30s	$-I_9$

Seeking for hard positions (3/4)

N	d_N	T	B_N
10	18	1m	$-I_{10}$
11	19	2m	$-I_{11}$ Y_{11} J_{11}
12	21	4m	$-I_{12}$
13	22	6m	$-I_{13}$ Y_{13} J_{13}
14	23	30m	$-I_{14}$ Y_{14} $H_{14,4}$ J_{14}
15	25	1h	Y_{15} J_{15}
16	26	1h20	$-I_{16}$ $H_{16,4}$ J_{16}
17	28	3h	$-I_{17}$

Seeking for hard positions (4/4)

N	d_N(*)	T	B_N
18	29	5h	$-I_{18}$ Y_{18} J_{18} $H_{18,4}$
19	30	16h	$-I_{19}$ Y_{19} J_{19} $H_{19,4}$ $H_{19,8}$ $H_{19,10}$ $H_{19,2^{18}+2}$ $H_{19,2^{18}+4}$ $H_{19,2^{18}+8}$
20	32	4d	$-I_{20}$ $H_{20,8}$ J_{20}
21	33	8d	$-I_{21}$ Y_{21} J_{21} $H_{21,4}$ $H_{21,16}$ $H_{21,18}$ $H_{21,20}$ $H_{21,2^{20}+2}$ $H_{21,2^{20}+4}$ $H_{21,2^{20}+16}$
22	35	>25d	$-I_{22}$ Y_{22} J_{22} ...

Speeding up IDA*

- Make use of breakpoints and anti-adjacencies
- H_A = #bp
- H_B = #bp + $\lfloor \lambda$ #aa$\rfloor$
- λ such that IDA* using H_B remains optimal …
- … but faster than IDA* using H_A

End of part one

Part one : optimal solving

- Heuristic Search

Part two : approximate solving

- Monte-Carlo Search

R-approximation

- Sub-optimal and polynomial time algorithm A
- $L_A(p)$: length of a solution of problem p obtained with algorithm A
- $L^*(p)$: length of an optimal solution to problem p
- $R_A(p) = L_A(p) / L^*(p)$
- R-approximation(A) = $\max_p R_A(p)$
- Theoretical and worst case analysis
- Unburnt pancakes :
  - (Fischer 2005) R=2
- Burnt pancakes :
  - (Cohen Blum 1995) R=2

Experimental Optimality Ratios

- In practice, average case analysis
- $L^*(p)$ is unknown and replaced by the best lower bound known so far
- $L^*(p) = \#bp$
- Experimental Optimality Ratio (EOR)
- $EOR_A(p) = L_A(p) / \#bp$
- $EOR(A) = \text{mean}_p\ EOR_A(p)$
- Unburnt pancakes :
  - (Fischer 2005) EOR = 1.22
  - (Bouzy 2015) EOR = 1.04

Experimental Optimality Ratios

- Nested Monte Carlo Search (NMCS) (Cazenave 2009)
- Simulations == (Cohen and Blum 1995)
- L*(p) = #bp   calibration IDA* optimal but EOR(IDA*) ≈ 1.2

		NMCS + Cohen Blum		
N	L	EOR	T	level
8	10.0	1.33	0.01s	2
8	10.0	1.33	0.02s	3
16	20.3	1.31	0.03s	2
16	19.9	1.28	2s	3
32	40.7	1.29	1.2s	2
64	122	1.91	0s	0
64	98	1.53	0.05s	1
128	250	1.95	0s	0
128	203	1.59	0.62s	1
256	505	1.98	0.01s	0

Conclusions and future work

- The burnt pancake problem
  - New lower bounds on the diameter with $g(-I_n)$
  - New hard stacks $H_{n,m}$
  - IDA* and preliminary heuristic function
  - EOR with NMCS + Cohen & Blum algorithm as simulator

- Future work :
  - Burnt pancake problem :
    - IDA* and an admissible heuristic function including anti-adjacency
  - Unburnt pancake problem
    - Find new results on the diameter
    - Generate complex stacks

Thank you for your attention!

Questions ?

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