ON THE COEFFICIENT OF RACIAL LIKENESS.

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EVERY craniologist and indeed every physical anthropologist has come up against the difficulty of comparing two races of which it is only possible to secure a limited number of individuals of one or other or both races. Not unnaturally he is driven under the circumstances to seek help by measuring a large number of characters in order to compensate for few individuals*. We have frequently to admit that relatively few individuals are available in many anthropometric inquiries, and that we really must compensate for the smallness of our sample by the largeness of our character series. But how is this to be done? We can compare the means for our two small groups character by character, and if we are trained statisticians we shall compare these mean differences with their standard deviations. But when a considerable number of the characters do not show differences markedly significant with regard to their probable errors, we are left in considerable doubt as to what inference may be safely drawn from the whole series. We need a single numerical measure of the whole system of differences, something which will express by a single coefficient the measure of resemblance (or divergence) of the two races or groups. Such a measure or coefficient I term a *Coefficient of Racial Likeness* (C.R.L.). It should be a measure, not of how far the two races or tribes are alike or divergent, but of *how far on the given data* we can assert significant resemblance or divergence.

Let us suppose m_s to be the mean in the first group of the sth character, σ_s its standard deviation and n the size of the sample; let m_s', σ_s' and n' be the corresponding quantities for the second sample. Then the difference of the means will be $m_s - m_s'$ and, supposing as will be the fact for proper random sampling that m_s is not correlated with m_s', the standard deviation of the difference will be $\sqrt{\frac{\sigma_s^2}{n} + \frac{\sigma_s'^2}{n'}}$. Similarly for a second character t we have to compare $m_t - m_t'$ with

$$\sqrt{\frac{\sigma_t^2}{n} + \frac{\sigma_t'^2}{n'}}.$$

Now if we are really taking samples from the *same* population, the mean of all m_s's and of all m_s''s will be the same, or the mean value of $m_s - m_s'$ or of $m_t - m_t'$ will be zero. Further, the distribution of difference of means will be like the dis-

* I have known cases in which an anthropologist has measured 50 to 100 characters in the 20 to 30 individuals of one tribe and sex, who were accessible to him, and he has pressed me to tell him whether this group was distinguishable from a similar small sample of a second tribe. Thus the problem is a very real one.

tribution of means themselves (if there be no association of the samples), closely normal whatever the original population may be, and will be the more rapidly normal as the size of the sample increases if the original population be, as it actually is in most anthropometric series, approximately normal. Accordingly each series like $m_s - m_s'$ will be given by a normal curve with S.D. equal to $\sqrt{\frac{\sigma_s^2}{n} + \frac{\sigma_s'^2}{n'}}$. All these normal curves will be reduced to one and the same scale if we take as our variate

$$X_s = (m_s - m_s') \Big/ \sqrt{\frac{\sigma_s^2}{n} + \frac{\sigma_s'^2}{n'}},$$

i.e. the variation of the X_s's will be about the origin of a normal curve with standard deviation unity.

If then we could consider X_s, X_t, etc. as independent characters, each would be a random drawing from a normal population of standard deviation unity; and if we took M, such drawings

$$\Sigma = \left\{\frac{1}{M} S \left(\frac{(m_s - m_s')^2}{\frac{\sigma_s^2}{n} + \frac{\sigma_s'^2}{n'}}\right)\right\}^{\frac{1}{2}}$$

should within the error of random sampling approach the value unity.

And further

$$\Sigma^2 = \left\{\frac{1}{M} S \left(\frac{(m_s - m_s')^2}{\frac{\sigma_s^2}{n} + \frac{\sigma_s'^2}{n'}}\right)\right\}$$

should also approach the value unity. We may adopt either of these values we please, but we shall have a difference in the probable error of our result according to our choice of Σ or Σ^2 to be dealt with.

Now what we are really doing here is to sample from a variate X (i.e. values $X_1, X_2, \ldots X_s, \ldots X_t, \ldots$), which is distributed normally, and determining its standard deviation or its standard deviation squared. The distribution of both of these are known and accordingly their probable values, mean values and standard deviations.

The curve for the distribution of Σ is

$$y = y_0 \Sigma^{M-2} e^{-\frac{1}{2}M\Sigma^2},$$

and the most probable value of Σ is

$$\tilde{\Sigma} = 1 - \frac{2}{M},$$

while the standard deviation of Σ is

$$\sigma_\Sigma = \sqrt{\frac{2}{M}} \left(\frac{M-1}{2} - \left\{\frac{\Gamma(\frac{1}{2}M)}{\Gamma(\frac{1}{2}(M-1))}\right\}^2\right)^{\frac{1}{2}}$$

$$= \frac{1}{\sqrt{2M}}\left(1 - \frac{1}{8M}\right),$$

approximately.

If M be large it will be adequate to suppose the distribution of Σ to be normal; for practical purposes this may be supposed reached by $M = 20$*, in which case we may take $\sigma_{\Sigma} = \frac{1}{\sqrt{2M}}$ and represent our result as

$$(\text{C.R.L.})_1 = \left\{ S\left(\frac{1}{M}\frac{(m_s - m_s')^2}{\frac{\sigma_s^2}{n} + \frac{\sigma_s'^2}{n'}}\right)\right\}^{\frac{1}{2}} - 1 \left[+\frac{2}{M}\right] \pm \frac{\cdot 67449}{\sqrt{2M}} \quad \text{.........(i)},$$

the value of C.R.L. varying round zero, if the two groups are from the same race with the probable error $\cdot 67449/\sqrt{2M}$. As a rule the races are sufficiently divergent to make the term $\left[+\frac{2}{M}\right]$ of small importance; for twenty characters it only contributes ·1, while it is the whole digits which have really to be considered.

If we prefer to deal with Σ^2 instead of Σ, the curve for its distribution is

$$y = y_0 (\Sigma^2)^{\frac{M-3}{2}} e^{-\frac{1}{2}M\Sigma^2}.$$

Here the most probable value of Σ^2 is

$$(\tilde{\Sigma}^2) = 1 - \frac{1}{M},$$

and the standard error σ_{Σ^2} is given by

$$\sigma_{\Sigma^2} = \sqrt{\frac{2}{M}}\left(1 - \frac{1}{2M}\right),$$

approximately.

Accordingly:

$$(\text{C.R.L.})_2 = S\left\{\frac{1}{M}\left(\frac{(m_s - m_s')^2}{\frac{\sigma_s^2}{n} + \frac{\sigma_s'^2}{n'}}\right)\right\} - 1 \left[+\frac{1}{M}\right] \pm \cdot 67449\sqrt{\frac{2}{M}} \quad \text{......(ii)},$$

where the value of C.R.L. will vary round zero with a probable error of

$$\cdot 67449\sqrt{\frac{2}{M}}.$$

Either value of C.R.L. might be taken as our standard measure of racial resemblance, and I considered both in 1919, and preferred the second, because the term in square brackets could be more frequently neglected. But unfortunately it appeared in *Biometrika*, Vol. XIII. p. 248, with the probable error of (i), i.e. $\frac{\cdot 67449}{\sqrt{2M}}$, instead of (ii), i.e. $\cdot 67449\sqrt{\frac{2}{M}}$, and this slip has been perpetuated in craniometric papers since. I did not notice the slip till reading through the proofs of Miss Hooke's paper in the current number. The slip is corrected in that paper and also in those of Mr Morant, and of Mr Morant and Miss Hooke in this issue.

* See Table, *Biometrika*, Vol. X. p. 529.

In the papers on the Tibetan*, Nepalese† and Egyptian‡ crania by Mr Morant and on the Burmese crania by Miss Tildesley§, the probable errors of the C.R.L. require *doubling*. This error, however, makes no difference in the conclusions drawn, for the probable error is merely intended to enable an appreciation to be made of how far the intensity of the coefficient is influenced by random sampling and how far by the fact that the two groups compared belong to markedly different races. In nearly all cases dealt with, even in comparing English with English, it is seen at once that the coefficient is influenced in the first place by the differential characterization and not by the sampling, which is of a quite secondary order.

We must halt, however, here to remark on another important point. σ_s and σ_s' are supposed to be the standard deviations of the sth character in the two populations of which our groups are respectively samples. They do not therefore vary with the sample or contribute to the probable error of C.R.L. But unfortunately we do not know σ_s and σ_s' and if we determine them from our samples, which *à priori* are supposed somewhat small, they will have large probable errors. Indeed the determination of variability from small samples constantly leads to larger divergences between the variability in the two samples than exists between the variabilities of two different races based on adequate numbers. For this reason I concluded that it would be unwise to use the values of σ_s and σ_s' derived from the small samples themselves, but that it would, having regard to the fact that the different races of men are not widely divergent in variability, be best to use the system of standard deviations obtained from large numbers, rather than those from the small samples under immediate consideration. For this purpose I selected the 1700 Dynastic Egyptian skulls which had been measured in the Laboratory and gave reasonable values of the standard deviations based on 700 to 800 crania of each sex. The formula then simplifies to

$$\text{C.R.L.} = S\left(\frac{1}{M}\frac{nn'}{n+n'}\frac{(m_s - m_s')^2}{\sigma_s^2}\right) - 1\left[+\frac{1}{M}\right] \pm \cdot 67449\sqrt{\frac{2}{M}},$$

where σ_s^2 is the variance of the sth character in the standard population.

If the series of either group be sufficiently long, as in the case of the Farringdon Street English, then we may use the σ_s's as found from it, but for the special case for which the C.R.L. was devised, where n and n' are relatively small, and this smallness is to be compensated by measuring many characters M, this is unwise. It is better, I think, to use the idea of a general human variability, slightly modified from one race to a second, than to increase the random errors of our coefficient by making crude approximations to the variabilities of the unknown populations from our small samples themselves.

Another and most important aspect of the matter now arises for consideration. Let us suppose the means of the sth character in the two sampled populations to

* *Biometrika*, Vol. XIV. pp. 207 *et seq.*

† *Biometrika*, Vol. XVI. pp. 54—73.

‡ *Biometrika*, Vol. XVII. pp. 1—52.

§ *Biometrika*, Vol. XIII. pp. 248—351.

be $\overline{m}_s$ and $\overline{m}_s'$ respectively. We can write $m_s = \overline{m}_s + \delta\overline{m}_s$ and $m_s' = \overline{m}_s' + \delta\overline{m}_s'$, where $\delta\overline{m}_s$ and $\delta\overline{m}_s'$ are statistical differences and not infinitely small mathematical differentials. Accordingly we have

$$\begin{aligned}\text{C.R.L.} = S&\left\{\frac{1}{M}\frac{nn'}{n+n'}\cdot\frac{(\overline{m}_s-\overline{m}_s')^2}{\sigma_s^2}\right\}\\ &+ 2S\frac{nn'}{M}\left\{\frac{(\overline{m}_s-\overline{m}_s')}{(n+n')\,\sigma_s^2}\right\}\delta\overline{m}_s\\ &- 2S\left\{\frac{nn'}{M}\frac{\overline{m}_s-\overline{m}_s'}{(n+n')\,\sigma_s^2}\right\}\delta\overline{m}_s'\\ &+ S\left\{\frac{1}{M}\frac{nn'}{n+n'}\frac{(\delta\overline{m}_s)^2}{\sigma_s^2}\right\} + S\left\{\frac{1}{M}\frac{nn'}{n+n'}\frac{(\delta\overline{m}_s')^2}{\sigma_s^2}\right\} - 2S\left\{\frac{1}{M}\frac{nn'}{n+n'}\right\}\delta\overline{m}_s\,\delta\overline{m}_s' - 1.\end{aligned}$$

Now when we take the mean value of all these summations for a large number of samples, the first is a constant and does not vary, the second and third vanish because the mean value of $\delta\overline{m}_s$ and $\delta\overline{m}_s'$ is zero, the mean value of $(\delta\overline{m}_s)^2$ is σ_s^2/n and of $(\delta\overline{m}_s')^2$ is σ_s^2/n', while lastly the mean value of $\delta\overline{m}_s\,\delta\overline{m}_s'$ is zero, for they are uncorrelated. Thus the mean value of the terms in $(\delta\overline{m}_s)^2$ and $(\delta\overline{m}_s')^2$ is $S\left(\frac{1}{M}\right)=1$, which cancels with the -1, and we see that the main component of the C.R.L. is the term $S\left\{\frac{1}{M}\frac{nn'}{n+n'}\frac{(\overline{m}_s-\overline{m}_s')^2}{\sigma_s^2}\right\}$, that is it is determined by the difference in the means of the sampled populations. It is really these $\delta\overline{m}_s$, $\delta\overline{m}_s'$ which give rise to the variability of the C.R.L. when we take continuous pairs of samples, and which is expressed by the probable error. It is thus not likely, unless the means are the same for all corresponding characters in the two sampled populations, that the C.R.L. will be zero. It is easily shown from a slight experience in comparing data for various races that the variations expressed in $\delta\overline{m}_s$, $\delta\overline{m}_s'$ (σ_s^2 being considered constant) contribute little to the value of the C.R.L.; that depends for its intensity on the fundamental term

$$S\left\{\frac{1}{M}\frac{nn'}{n+n'}\frac{(\overline{m}_s-\overline{m}_s')^2}{\sigma_s^2}\right\}.$$

The question may of course be raised, whether another and better expression might not be found as a coefficient of racial likeness. It may be quite well argued that product terms ought to be introduced into its expression. We know that while m_s is not correlated with m_s', yet m_s is correlated with m_t, in fact if we are taking a series of samples from the same two populations the correlation between m_s and m_t is r_{st} and between m_s' and m_t' will be r_{st}', where r_{st} and r_{st}' are the correlations of the sth and tth characters in the two populations themselves*. Now it would not be unreasonable to suppose that just as we have taken $\sigma_s = \sigma_s'$ we may also take approximately $r_{st} = r_{st}'$. In this case it is easily shown that

$$\frac{nn'}{n+n'}\frac{(m_s-m_s')(m_t-m_t')}{\sigma_s\sigma_t}$$

* The correlation coefficient of the mean of two variates is the same as the correlation coefficient of the two variates themselves.

takes for its mean value

$$\frac{nn'}{n+n'}\frac{(\overline{m}_s-\overline{m}_s')(\overline{m}_t-\overline{m}_t')}{\sigma_s\sigma_t}+r_{st}.$$

As in the case of $\frac{nn'}{n+n'}\frac{(m_s-m_s')^2}{\sigma_s^2}$ it will probably be the first term in this expression which provides the chief contribution. Although some characters of the skull are fairly highly correlated, others have hardly any correlation at all, and some may have positive and some negative signs.

If, however, we start with our variates

$$X_s=\sqrt{\frac{nn'}{n+n'}}\frac{m_s-m_s'}{\sigma_s},\quad X_t=\sqrt{\frac{nn'}{n+n'}}\frac{m_t-m_t'}{\sigma_t},$$

and *suppose our two samples to belong to the same population*, then we have $\sigma_{X_s}=1$, $\sigma_{X_t}=1$ and $r_{X_sX_t}=r_{st}$. Hence if we form the determinant

$$R=\begin{vmatrix} 1, & r_{12}, & \dots\dots\dots & r_{1M} \\ r_{21}, & 1, & \dots\dots\dots & r_{2M} \\ \dots & \dots & \dots\dots\dots & \dots \\ \dots & \dots & \dots\dots\dots & \dots \\ r_{M1}, & r_{M2} & \dots\dots\dots & 1 \end{vmatrix}$$

and its minors, we should have

$$\chi^2=\frac{1}{R}\left\{\overset{M}{\underset{1}{S}}(R_{ss}X_s^2)+2S'(R_{st}X_sX_t)\right\},$$

and our distribution surface with an element $z_0e^{-\frac{1}{2}\chi^2}\,dX_1\,dX_2\dots dX_M$ of frequency. This would lead us to a χ^2, P test of the improbability of the material of the two groups being drawn from the same population, which might well be better theoretically than the C.R.L. discussed previously. Now it will be seen that in order to get the value of R and its minors we must know the correlations of M characters in a standard population. The ideal value of M would be 40 to 50, this would involve the calculation of 780 or 1225 correlation coefficients. The largest number of correlation coefficients yet computed are those for the Egyptian 26th to 30th Dynastic series. But in that case after very strenuous labour only 312 correlations were found. Thus, if the particular characters there chosen had been so chosen as to give a closed series, i.e. if 24 or 25 characters had been selected and correlated only among themselves, we could only take $M=24$ or 25 and must *always* take the same 24 or 25 characters, whatever pairs of races were being compared. This is an impossibility unless all craniologists agree to consider the same standard characters. But suppose this were done, and that there were 30 to 40 standard characters and the 435 to 780 correlations were all known, then indeed the real difficulty of the task would begin, we should have to compute a series of determinants, 780 to 820 in number, each consisting of 30 to 40 rows and columns. The task would be gigantic and if completed would be of no service should a series of crania be measured in which even one of the standard characters had been omitted. For the statistician, as for the statesman, the ideally best is not always the wisest course. Even if these coefficients could be computed, we should have to deal with the determining and adding together of 465 (30 + 435) to 820 (40 + 780) terms instead of the 30 to 40 terms of the present cruder coefficient. Further, while some of these terms are positive others are negative, and the great bulk, although by no means all, are very small*. Hence the 435 to 780 terms may not contribute as much as we might possibly anticipate to the 465 to 820 terms suggested for our χ^2. The mean correlation of the characters of the skull—without regard to sign—is only about ·3, and would be considerably less paying attention to sign. Still the fundamental weakness of the Coefficient of Racial Likeness lies in the fact that it neglects the correlations between the characters dealt with. If we examine such determinantal relations as R_{ss}/R and R_{st}/R, we see that

$$R_{ss}/R = 1 - \text{squares and higher powers and products of correlations,}$$
$$R_{st}/R = r_{st} - \text{products and higher powers and products of correlations,}$$
$$= 1 - \Delta_{ss} \text{ and } r_{st} - \Delta_{st}, \text{ say, respectively.}$$

I have found it possible to express Δ_{ss} and Δ_{st} as approximate functions of *mean* correlation values, but I have not been able to determine how close this approximation is in the case of 15 to 20 rowed determinantal ratios without numerical experiment, the labour of which would be very great. But were these approximate expressions adequate in the case of cranial correlations, I cannot conceive that any craniologist could at present be induced to calculate the hundreds of correlations requisite (i.e. the r_{st}'s), or having found them to compute the hundreds of terms requisite to determine χ^2.

However, I do see an entirely different method of approaching the subject when once we have 50 to 100 cranial series, each containing 50 to 100 individuals of one sex measured in a standardised manner. But that method is for co-operative work in the future.

Meanwhile the C.R.L. as now in use seems to me the best test available, if used with discretion, i.e. tested in male and female series, and for indices and angles, as well as for all characters, and compared in its results with conclusions drawn from the correspondence or divergence of the mean racial cranial contours†. If any one has a sounder coefficient to propose, I shall not be the last to welcome and use it.

Assuming, however, that the theoretical difficulties of the C.R.L. can be disregarded, and that it can be looked upon as *practically* an approximate measure of racial association, if not an ideally adequate one, we may ask: how has it fulfilled this purpose? Does it give on the whole a rough measure of racial likeness, when we classify races by the general impressions which anthropologists have hitherto adopted rather than by accurate numerical relations? On the whole, while it contradicts some current anthropological beliefs, and suggests some

* See, on all these points, the paper by Pearson and Davin, "Biometric Constants of the Human Skull," *Biometrika*, Vol. XVI. pp. 347—363.

† A single number to represent the degree of resemblance between two mean racial cranial contours is badly needed.

hitherto unsuspected relationships, it does not give results wildly discordant with the beliefs and impressions of anthropology. It seems rather to confirm, to extend and in special cases to correct them.

Up to the present nearly 760 C.R.L.'s have been determined by the Biometric School of craniometry. It was considered originally possible that there might be a resemblance in shape between two races, when there failed to be a resemblance in absolute size. For this reason C.R.L.'s were worked out for indices and angles only; of these we have 340, while for absolute measurements, angles and indices, i.e. for characters of all types combined, we have 417 cases. It was actually found that in certain cases there was a greater resemblance in shape than size, but it may be doubted whether it is worth while to separate size and shape characters, as this of course lessens the total number of characters available for computing the coefficient. We prefer to use two series only, one for shape characters and one for *all* characters.

The following table gives the distribution of coefficients found:

Values of C.R.L.'s.

	Less than *1* ...	*1—4*	*4—7*	*7—10*	*10—13*	*13—16*	*16—19*	*19—22*	*22—25*	*25—28*	*28—31*	Above *31*	Totals
All Characters	54	131	73	39	26	23	15	12	8	11	2	23	417
Indices and Angles	56	106	59	35	19	16	9	12	4	6	2	16	340
Totals	110	237	132	74	45	39	24	24	12	17	4	39	757

Of course the main object of the biometric inquirers was to find resemblances, not to search for widely divergent races. Hence no special stress is to be laid on the frequency distributions, but knowing the races involved in each group of values it seemed possible to arrange a classification giving five grades of association and seven grades of divergence, and after stating these—at any rate as provisional terms—we will then consider what pairs of races fall into some of these categories.

Degrees of Association			Degrees of Divergence		
Grade	Range	Class	Grade	Range	Class
I	Less than *1*	Very intimate Association	I	*13—16*	Slight Divergence
II	*1— 4*	Close Association ..	II	*16—19*	Moderate Divergence
III	*4— 7*	Moderate Association ...	III	*19—22*	Moderate Divergence
IV	*7—10*	Slight Association ...	IV	*22—25*	Marked Divergence
V	*10—13*	Doubtful Association ...	V	*25—28*	Marked Divergence
—	—	—	VI	*28—31*	Marked Divergence
—	—	—	VII	Over *31*	Very wide Divergence

Of course, assuming the origin of man to be monogenetic, "Association" and "Divergence" are only relative terms of a continuous grade of relationship, indicating only the greater length of differentiated ancestry. But they are convenient terms. It is proper to look upon Anglo-Saxons and modern English as associated races, but on Chinese and English as divergent races, if we only mean by this that the forerunners of Chinese and English diverged much earlier from a common ancestry than English and Anglo-Saxons. To fix the limit of association and divergency at a C.R.L. = 13 is of course arbitrary, but it is convenient for practical purposes. It signifies that it would be difficult to place two races with this coefficient in the same family of races.

I will examine individually first some of the pairs which fall into my "Very wide Divergence" category. They are, considering only all characters:

The Dravidian Race as represented by the Maravar with Malays (31), with Burmese (38), with English (Whitechapel) (43), with Moriori (49), and with Aino (55). We should *à priori* probably have asserted all these races to be widely divergent, but in this marked divergence that the Malays and Burmese should be less marked is satisfactory, if again it be what some might anticipate*.

The Aino race with Tibetans (31), with Hindus (32), with Malayans (33), with Moriori (33), with Burmese (34), with English (35), with Nepalese (38), with Dravidians (55), and with Altai Telengites (71)†.

English 17th Century with Nepalese (35), with Burmans (46), and with Malays (77); the Prehistoric Egyptians (Naqada) with Burmans (55), and with Malayans (65); the Moriori with the Hindus (32) and with the Nepalese (33) are all self-explanatory. We are dealing with races which every one admits to be widely divergent. In practically all these cases our results are confirmed, if we limit our characters to indices and angles only. There are indeed some cases in which for shape only the divergence is more conspicuous than when we deal with both size and shape. Thus:

Eskimos with Maori (32), with Fuegians (35) with Moriori (51), with Aino (64); while for both size and shape the corresponding values are: (13), (20), (25) and (26) respectively. These connote rather considerable divergence but not so great as for shape only. We get measures of wide divergence if we deal with shape only in the case of the Malayans, e.g. with Aino (37), with Moriori (37), with Prehistoric Egyptians (110), and with 17th Century English (126). In all these cases we are dealing with what are admittedly widely divergent races. A very limited craniological experience would enable anyone to distinguish without computing a C.R.L. between the skulls of the last races. But we have cited the values here to show that the C.R.L. is a real criterion of cranial divergence.

It would not be fair, however, to pass by two remarkable values wherein the C.R.L. appears at first to fail. The skulls at Hythe have been measured to the

* It is easy to find races closely allied to this Dravidian stock, thus with Bengal Hindus (2), with Veddahs (2), with Andamanese (5), with Nepalese (6·5), with Burmese Hybrids (7), and with Karens (10).

† Again it is easy to find out associated races: with Fuegians (7), with Japanese (8), with Maori (8), with Koreans (9), and with Northern Chinese (16).

number of 315 by Professor Parsons, unfortunately for very few characters. Only eight are available for a C.R.L. They have been compared with the 17th Century English (Farringdon Street) (83) and with the Anglo-Saxons (73). These numbers indicate either (i) that the Hythe crania are very widely divergent from English and Anglo-Saxon, or (ii) that Professor Parsons' methods of measurement are very widely divergent from those laid down by the international concordat. Until the Hythe crania are measured for a large variety of characters in the standardised manner it will be impossible to say what is the real significance of the above C.R.L.'s.

If we now turn to the other extreme of our scale "very intimate association," we find ourselves dealing with: (*a*) the same race sampled by two craniologists, (*b*) local varieties of the same race, or (*c*) the same race at different epochs of its existence. Of course all cases of (*a*), (*b*) and (*c*) do not fall into Grade I of Association, but it is difficult to find any pair in Grade I which we are certain embraces two craniologically distinct races. A few examples of each class will suffice: (*a*) Eskimos measured by Fürst and Hansen, and those measured by Hrdlička (− 0·90), two series of Chinese (0·59), Moriori—Scott's series and Thomson's series—(0·41), Trans-Himalayan and Cis-Himalayan Bhotias (− 0·34) Nepalese Central and Nepalese Eastern (− 0·45), etc. The last two cases probably might be classified under (*b*). (*b*) Tibetans and Trans-Himalayan Bhotias (0·48), Burmans and Burmese Hybrids (0·78), Annamese and Southern Chinese (0·01), Siamese and Annamese (0·14), Torgods and Kalmucks (0·89), etc. (*c*) 17th Century English (Whitechapel) and British Iron Age (0·38), 1st and 2nd Dynasty (Royal Tombs) and 18th—20th Dynasty (0·9), Ptolemaic Period and Roman Period Egyptians (0·72), 1st and 2nd Dynasty Royal Tombs and Roman Period Egyptians (0·65), Prehistoric Egyptians (Naqada) and Ptolemaic Period Egyptians (0·04), etc.

The above illustrations of the two extremes of "marked divergence" and "intimate association" prepare us for having confidence in the C.R.L. in the intermediate grades.

Once this confidence is won—and those who have examined the 750 coefficients already computed will find in them confirmation of many conclusions reached in other ways—we cannot reject straight off results which are novel or against impressions often based on no well-defined quantitative research. I may mention one or two of these results, which I believe should not be straight off rejected, but deserve full consideration.

(i) The Moriori are more closely related to the Fuegians (4·6) than to the Maori (8·5). This may indicate an early transfer from Antarctic lands to South America.

(ii) The Aino are more closely related to the Fuegians (6·7) than to the Japanese (8·1), or to the Koreans (8·9), or to the Northern Chinese (16·1). A study of "fringe" peoples by aid of the C.R.L. may lead us to new ideas on the passage of human racial waves over the whole earth's surface. The presence of Lemuroids in Borneo and Madagascar led to strange hypotheses, until the fossils of Lemuroids were found in Europe, Asia, and America.

(iii) The crania of modern Abyssinians of the Tigré District are as closely related to Dynastic Egyptians (1·2 to 3·7) as the Dynastic are to the Predynastic Egyptians. The ancient Egyptian type has therefore been preserved in more than one form to modern days.

(iv) The English skull is nearer to that of the men of the British Iron Age than to that of Anglo-Saxons. Thus

17th Century English		Men of Iron Age	Anglo-Saxons
Males	Whitechapel Crania ...	(0·38)	(2·98)
	Moorfields Crania ...	(1·55)	(4·88)
	Farringdon Street Crania	(2·82)	(5·27)
Females	Whitechapel Crania ...	(1·84)	(9·61)
	Moorfields Crania ...	(1·32)	(4·66)
	Farringdon Street Crania	(2·81)	(6·51)

It will be evident from these figures that though the Anglo-Saxon is associated with the English skull, it stands in nothing like so close a relationship as the skull of the Iron Age men. Nor indeed is the Anglo-Saxon closer to the English than the Long Barrow cranium as measured by Schuster, which gave with Whitechapel (3·7) and with Farringdon Street (5·3).

If the coefficient of racial likeness is to be trusted, the belief that the English are in the main Anglo-Saxons must be discarded. This does not mean that there is not association of a "moderate" kind with the Anglo-Saxons—it is closer than the "doubtful association" of Bavarians with Würtemberger (12·1) or than the "marked divergence" of English and French (24·5)—but it is not of the "close association" type which the English have with men of the Iron Age (Grade II of Association).

I have said enough to indicate that not only the C.R.L. can confirm current impressions, but that it can raise new and suggestive problems. With the work done in the Biometric Laboratory by Miss M. Tildesley, Mr Morant and Miss B. Hooke we are now in a position to state that the coefficient can be a serviceable tool in craniometric research. Such a statement was not possible when the coefficient only stood on a not wholly adequate theoretical basis, but the present practical basis of 750 computed coefficients, capable of being set against many accepted racial relations, has given it a sounder position, and this we owe entirely to their assiduous labour.

The change in the C.R.L. can be illustrated in reference to the number of characters dealt with by comparing a few cases in which it has been calculated for two series of characters on the same series of crania.

Thus we have:

Series compared	Number of Characters	
Naqada *A*, *Q* Graves with Naqada *B*, *T*, *R* Graves	14 (1·62)	31 (0·65)
Modern Abyssinian with El Kubanieh South Graves	14 (1·16)	23 (1·87)
Naqada *B*, *T*, *R* Graves with 1st Dynasty Royal Tombs	14 (2·59)	31 (2·37)
El Kubanieh South Graves with 18th—21st Dynasty Tombs ...	14 (3·51)	27 (2·65)
Naqada *A* and *Q* Graves with 1st Dynasty Royal Tombs ...	14 (9·68)	31 (6·36)

While the experienced craniometrician would lay no real stress on the differences in the C.R.L. occurring between those with long and those with short series of characters—especially as the numbers of crania involved were not large—it might appear as if the longer series of characters involved in general a smaller value of the C.R.L. To test this point I took out of the 417 coefficients for all types the 313 for which the number of characters used was directly stated, and formed a correlation table for size of coefficient and number of characters used. The resulting table was somewhat "lumpy" owing to necessity or preference leading the biometric workers to adopt certain groups of characters, but I think the final result may be relied on. It is:

Correlation of the C.R.L. and Number of Characters used

$$r = \cdot 0527 \pm \cdot 0380.$$

In other words there is no evidence that the coefficient of racial likeness is influenced in one direction by the number of characters adopted. I think this must really *à priori* be obvious, if no selection has been made of those characters which in the two races differ most or differ least from one another.

I next proceeded to consider what influence the number of individual crania dealt with had on the coefficient. Considering only coefficients based on all characters (i.e. absolute size and indices and angles), I obtained a correlation table of 580 entries, entering each coefficient twice, once for each race dealt with. The table was still more "lumpy" than the previous one, as the skull frequencies run from 6 to 885. The answer I found for the Correlation of the C.R.L. and Number of Crania used was

$$r = + \cdot 1270 \pm \cdot 0276,$$

i.e. there was a not very important correlation between the number of skulls used and the coefficient, the *greater* the number of skulls the *larger* the coefficient. As a matter of fact the influence of the size of the two samples is largely obscured by the variation of the ratio $(\overline{m}_s - \overline{m}_s')^2/\sigma_s^2$ (see our p. 109). Thus we see that the actual values of n and n', the numbers of individuals in the two series compared, is not so influential as might have been anticipated. It would undoubtedly be well if n and n' as well as the cranial characters selected could be standardised. We should then make far more rapid progress in placing the various races of man into a classified scheme and seeing more clearly the nature of human evolution.

What we need are, say 50 to 100 crania of each sex of each race, and then 40 to 50 characters measured in a standardised manner. I think the C.R.L., as it has been already used, forms a very good rough guide to racial association—or in some few cases, perhaps, to the extraordinary personal equations of certain craniologists—but if we had such series as I have suggested its value would be markedly emphasised. Owing to the steady measuring and tabling work of German and English investigators, such long series for an adequate number of characters are becoming greater in number and they will one day form a sound basis for a theory of racial evolution in man. Any argument from series of 6 to 10 crania—even using the C.R.L.—is, I think, to be deprecated*. It may be all that is feasible at present, but conclusions based on such series cannot be treated as final.

* I noted that out of the 580 coefficients tabled by me nearly a sixth, 94, were for series with less than 16 crania measured.