HOMOGRAPHIES
H_{3x3}

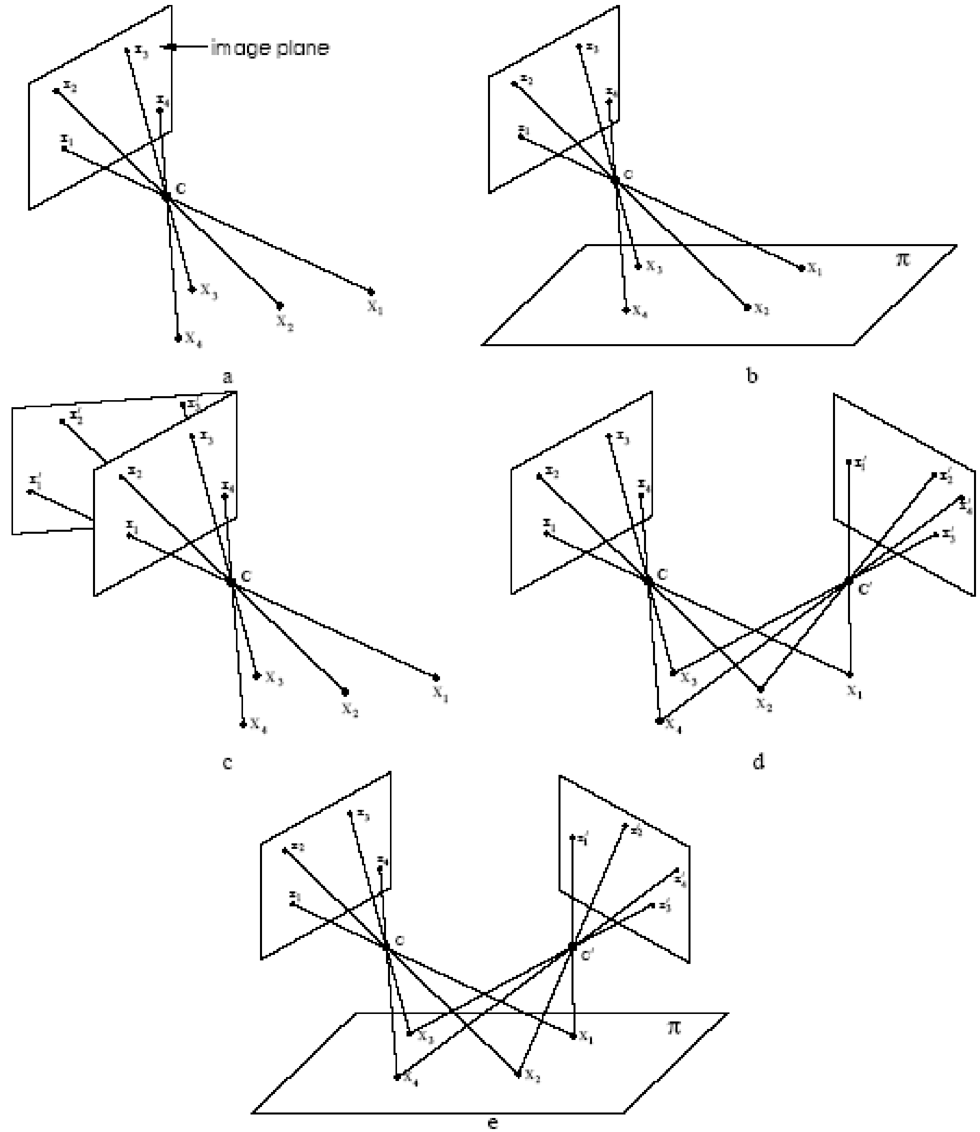
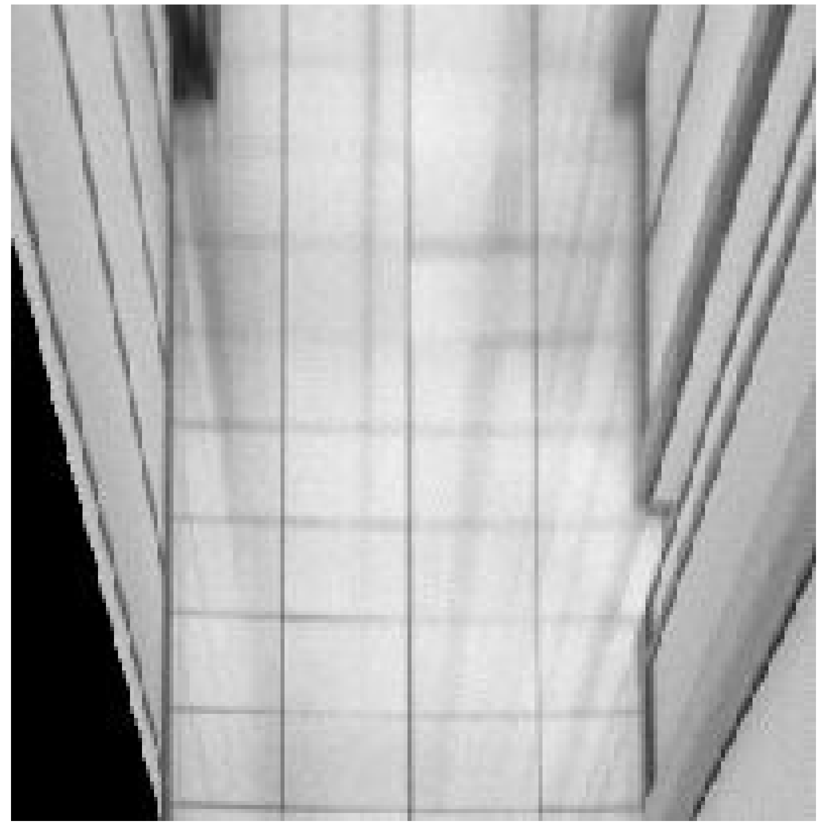
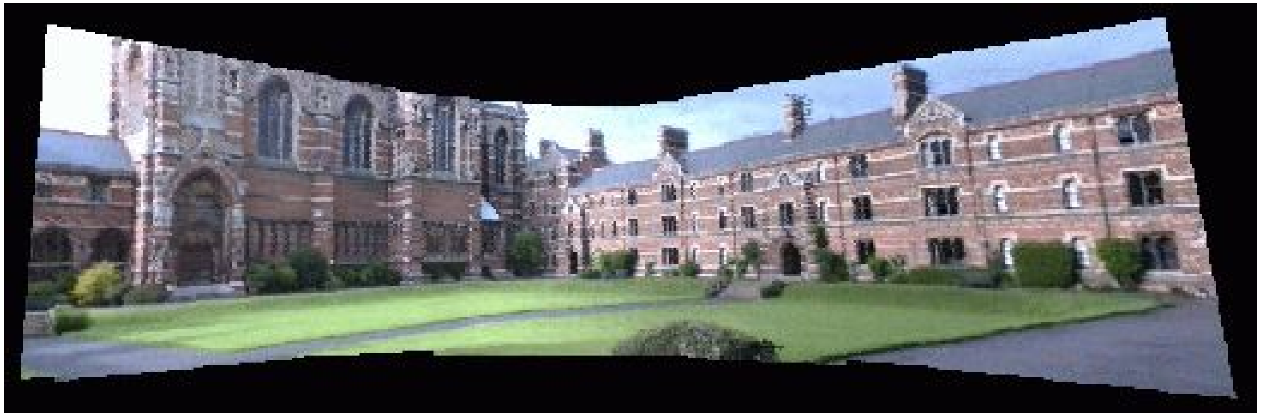


Fig. 1.1. **The camera centre is the essence.** *(a) Image formation: the image points* $\mathbf{x}_i$ *are the intersection of a plane with rays from the space points* $\mathbf{X}_i$ *through the camera centre* $\mathbf{C}$. *(b) If the space points are coplanar then there is a projective transformation between the world and image planes,* $\mathbf{x}_i = \mathrm{H}_{3\times3}\mathbf{X}_i$. *(c) All images with the same camera centre are related by a projective transformation,* $\mathbf{x}'_i = \mathrm{H}'_{3\times3}\mathbf{x}_i$. *Compare (b) and (c) – in both cases planes are mapped to one another by rays through a centre. In (b) the mapping is between a scene and image plane, in (c) between two image planes. (d) If the camera centre moves, then the images are in general not related by a projective transformation, unless (e) all the space points are coplanar.*

GENERER DES VUES SYNTHETIQUES



MOSAIQUER



Original vue dessus

Original vue oblique

Vue de dessus
redressée

Proposez un algorithme pour estimer
l'Homographie à appliquer

bjective

iven $n \geq 4$ 2D to 2D point correspondences $\{\mathbf{x}_i \leftrightarrow \mathbf{x}'_i\}$, determine the 2D homography atrix $\mathtt{H}$ such that $\mathbf{x}'_i = \mathtt{H}\mathbf{x}_i$.

lgorithm

(i) For each correspondence $\mathbf{x}_i \leftrightarrow \mathbf{x}'_i$ compute the matrix $\mathtt{A}_i$ from (4.1). Only the first two rows need be used in general.

(ii) Assemble the n 2×9 matrices $\mathtt{A}_i$ into a single $2n \times 9$ matrix $\mathtt{A}$.

(iii) Obtain the SVD of $\mathtt{A}$ (section A4.4(p585)). The unit singular vector corresponding to the smallest singular value is the solution $\mathbf{h}$. Specifically, if $\mathtt{A} = \mathtt{UDV}^\mathsf{T}$ with $\mathtt{D}$ diagonal with positive diagonal entries, arranged in descending order down the diagonal, then $\mathbf{h}$ is the last column of $\mathtt{V}$.

(iv) The matrix $\mathtt{H}$ is determined from $\mathbf{h}$ as in (4.2).

orithm 4.1. *The basic DLT for* $\mathtt{H}$ *(but see algorithm 4.2(*p*109) which includes normalization).*

$$\mathrm{H}\mathbf{x}_i = \begin{pmatrix} \mathbf{h}^{1\mathsf{T}}\mathbf{x}_i \\ \mathbf{h}^{2\mathsf{T}}\mathbf{x}_i \\ \mathbf{h}^{3\mathsf{T}}\mathbf{x}_i \end{pmatrix}.$$

Writing $\mathbf{x}'_i = (x'_i, y'_i, w'_i)^{\mathsf{T}}$, the cross product may then be given explicitly as

$$\mathbf{x}'_i \times \mathrm{H}\mathbf{x}_i = \begin{pmatrix} y'_i\mathbf{h}^{3\mathsf{T}}\mathbf{x}_i - w'_i\mathbf{h}^{2\mathsf{T}}\mathbf{x}_i \\ w'_i\mathbf{h}^{1\mathsf{T}}\mathbf{x}_i - x'_i\mathbf{h}^{3\mathsf{T}}\mathbf{x}_i \\ x'_i\mathbf{h}^{2\mathsf{T}}\mathbf{x}_i - y'_i\mathbf{h}^{1\mathsf{T}}\mathbf{x}_i \end{pmatrix}.$$

Since $\mathbf{h}^{j\mathsf{T}}\mathbf{x}_i = \mathbf{x}_i^{\mathsf{T}}\mathbf{h}^j$ for $j = 1, \ldots, 3$, this gives a set of three equations in the entries of H, which may be written in the form

$$\begin{bmatrix} \mathbf{0}^{\mathsf{T}} & -w'_i\mathbf{x}_i^{\mathsf{T}} & y'_i\mathbf{x}_i^{\mathsf{T}} \\ w'_i\mathbf{x}_i^{\mathsf{T}} & \mathbf{0}^{\mathsf{T}} & -x'_i\mathbf{x}_i^{\mathsf{T}} \\ -y'_i\mathbf{x}_i^{\mathsf{T}} & x'_i\mathbf{x}_i^{\mathsf{T}} & \mathbf{0}^{\mathsf{T}} \end{bmatrix} \begin{pmatrix} \mathbf{h}^1 \\ \mathbf{h}^2 \\ \mathbf{h}^3 \end{pmatrix} = \mathbf{0}. \tag{4.1}$$

These equations have the form $\mathrm{A}_i\mathbf{h} = \mathbf{0}$, where A_i is a 3×9 matrix, and $\mathbf{h}$ is a 9-vector made up of the entries of the matrix H,

$$\mathbf{h} = \begin{pmatrix} \mathbf{h}^1 \\ \mathbf{h}^2 \\ \mathbf{h}^3 \end{pmatrix}, \qquad \mathrm{H} = \begin{bmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ h_7 & h_8 & h_9 \end{bmatrix} \tag{4.2}$$

with h_i the i−th element of $\mathbf{h}$. Three remarks regarding these equations are in order here.

(i) The equation $\mathrm{A}_i\mathbf{h} = \mathbf{0}$ is an equation *linear* in the unknown $\mathbf{h}$. The matrix elements of A_i are quadratic in the known coordinates of the points.

(ii) Although there are three equations in (4.1), only two of them are linearly independent (since the third row is obtained, up to scale, from the sum of x'_i times the first row and y'_i times the second). Thus each point correspondence gives two equations in the entries of H. It is usual to omit the third equation in solving for H ([Sutherland-63]). Then (for future reference) the set of equations becomes

$$\begin{bmatrix} \mathbf{0}^{\mathsf{T}} & -w'_i\mathbf{x}_i^{\mathsf{T}} & y'_i\mathbf{x}_i^{\mathsf{T}} \\ w'_i\mathbf{x}_i^{\mathsf{T}} & \mathbf{0}^{\mathsf{T}} & -x'_i\mathbf{x}_i^{\mathsf{T}} \end{bmatrix} \begin{pmatrix} \mathbf{h}^1 \\ \mathbf{h}^2 \\ \mathbf{h}^3 \end{pmatrix} = \mathbf{0}. \tag{4.3}$$

This will be written

$$\mathrm{A}_i\mathbf{h} = \mathbf{0}$$

where A_i is now the 2×9 matrix of (4.3).

$$\begin{bmatrix} 0 & 0 & 0 & -x_iw'_i & -y_iw'_i & -w_iw'_i & x_iy'_i & y_iy'_i \\ x_iw'_i & y_iw'_i & w_iw'_i & 0 & 0 & 0 & -x_ix'_i & -y_ix'_i \end{bmatrix} \tilde{\mathbf{h}} = \begin{pmatrix} -w_iy'_i \\ w_ix'_i \end{pmatrix}$$

From