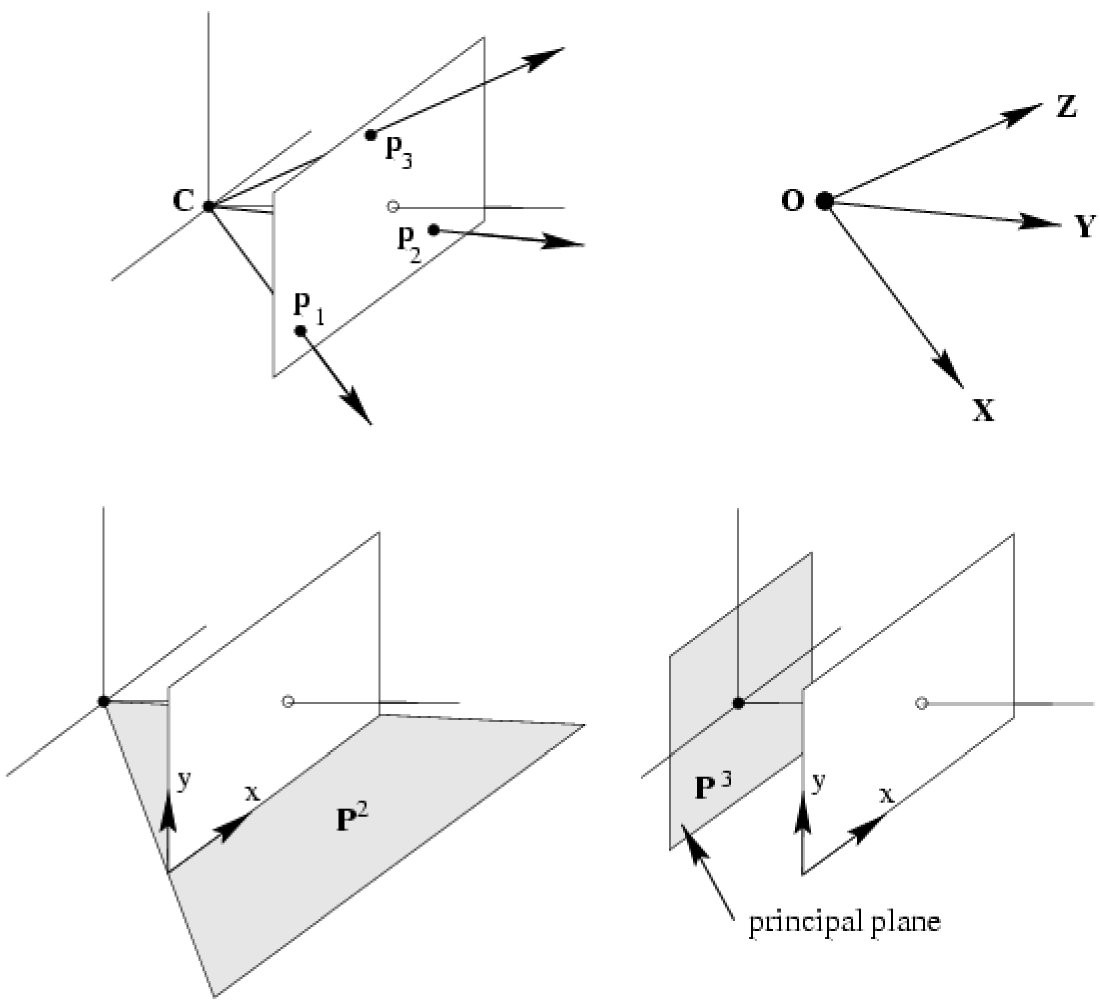
Anatomie de la caméra projective :

$$P_{3x4}=[M_{3x3}\ p_4]=[p_1\ p_2\ p_3\ p_4]=\begin{bmatrix} P^{1T} \\ P^{2T} \\ P^{3T} \end{bmatrix}$$

rang(P)=3 donc noyau de P de dimension 1 et PC=0=$(0,0,0)^T$, où C est le centre de la caméra en coordonnées homogènes, dont l'image est le seul point indéfini.



Camera centre. The camera centre is the 1-dimensional right null-space $\mathbf{C}$ of $\mathtt{P}$, i.e. $\mathtt{P}\mathbf{C} = \mathbf{0}$.

- ◇ **Finite camera** ($\mathtt{M}$ is not singular) $\mathbf{C} = \begin{pmatrix} -\mathtt{M}^{-1}\mathbf{p}_4 \\ 1 \end{pmatrix}$
- ◇ **Camera at infinity** ($\mathtt{M}$ is singular) $\mathbf{C} = \begin{pmatrix} \mathbf{d} \\ 0 \end{pmatrix}$ where $\mathbf{d}$ is the null 3-vector of $\mathtt{M}$, i.e. $\mathtt{M}\mathbf{d} = \mathbf{0}$.

Column points. For $i = 1, \ldots, 3$, the column vectors $\mathbf{p}_i$ are vanishing points in the image corresponding to the X, Y and Z axes respectively. Column $\mathbf{p}_4$ is the image of the coordinate origin.

Principal plane. The principal plane of the camera is $\mathbf{P}^3$, the last row of $\mathtt{P}$.

Axis planes. The planes $\mathbf{P}^1$ and $\mathbf{P}^2$ (the first and second rows of $\mathtt{P}$) represent planes in space through the camera centre, corresponding to points that map to the image lines $x = 0$ and $y = 0$ respectively.

Principal point. The image point $\mathbf{x}_0 = \mathtt{M}\mathbf{m}^3$ is the principal point of the camera, where $\mathbf{m}^{3\mathsf{T}}$ is the third row of $\mathtt{M}$.

Principal ray. The principal ray (axis) of the camera is the ray passing through the camera centre $\mathbf{C}$ with direction vector $\mathbf{m}^{3\mathsf{T}}$. The principal axis vector $\mathbf{v} = \det(\mathtt{M})\mathbf{m}^3$ is directed towards the front of the camera.

Table 6.1. *Summary of the properties of a projective camera* $\mathtt{P}$. *The matrix is represented by the b[...]* *form* $\mathtt{P} = [\mathtt{M} \mid \mathbf{p}_4]$.

Les vecteurs colonnes = images du repère monde, axes et centre (p_4)

Le plan principal = plan dont l'image est la droite à l'infini du plan image, soit /PX=(x,y,0), c'est P^{3T}

P^1 = (C,(ΩY)) et P^2 =(C,(ΩX)) sont moins liés à la nature de la caméra qu'aux systèmes d'axes de l'image. Ainsi P1 Inter P2 n'est pas l'axe optique en général.

Le point principal est Axe principal Inter Plan Image et Axe Principal = (C, $(p_{31},p_{32},p_{33},0)^T$) donc Pont principal = $P(p_{31},p_{32},p_{33},0)^T$

Action de la matrice sur les points appartenant au plan à l'infini dont l'image représente les points de fuite. Seul M agit.

Si C=(X,Y,Z,T), on a X=det([p_2,p_3,p_4]),Y=-det([p_1,p_3,p_4]),Z=det([p_1,p_2,p_4]),T=-det([p_1,p_2,p_3]),

On peut trouver K et R par la décomposition RQ, où Q est triangulaire supérieure et R orthogonale

$$P_{3x4}=[M_{3x3}-M\tilde{C}]=K[R-R\tilde{C}]$$

Si on prend une image d'une image (homographie 2D), on a HPC=0 ssi PC=0 donc le centre apparent de la caméra n'est pas modifié

$$P_{3x4} = [3x3\, homgraphie]\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}[4x4\, homographie]$$

Si

$$P_{3x4} = \begin{bmatrix} 3.5e+2 & 3.4e+2 & 2.7e+2 & -1.4e+6 \\ -1.0e+2 & 2.3e+1 & 4.6e+2 & -6.3e+5 \\ 7.0e-1 & -3.5e-1 & 6.1e-1 & -9.2e+2 \end{bmatrix}$$

Alors

$$M = KR = \begin{bmatrix} 468.2 & 91.2 & 300.0 \\ 0 & 427.2 & 200.0 \\ 0 & 0 & 1.0 \end{bmatrix}\begin{bmatrix} 0.41 & 0.90 & 0.04 \\ -0.57 & 0.22 & 0.79 \\ 0.7 & -0.35 & 0.61 \end{bmatrix}$$

$$\tilde{C} = (1000.0,\ 2000.0,\ 1500.0)^{T}$$

Objective

Given $n \geq 6$ world to image point correspondences $\{\mathbf{X}_i \leftrightarrow \mathbf{x}_i\}$, determine the Maximum Likelihood estimate of the camera projection matrix P, i.e. the P which minimizes $\sum_i d(\mathbf{x}_i, \mathtt{P}\mathbf{X}_i)^2$.

Algorithm

(i) **Linear solution.** Compute an initial estimate of P using a linear method such as algorithm 4.2(*p*109):

(a) **Normalization:** Use a similarity transformation T to normalize the image points, and a second similarity transformation U to normalize the space points. Suppose the normalized image points are $\tilde{\mathbf{x}}_i = \mathtt{T}\mathbf{x}_i$, and the normalized space points are $\tilde{\mathbf{X}}_i = \mathtt{U}\mathbf{X}_i$.

(b) **DLT:** Form the $2n \times 12$ matrix A by stacking the equations (7.2) generated by each correspondence $\tilde{\mathbf{X}}_i \leftrightarrow \tilde{\mathbf{x}}_i$. Write $\mathbf{p}$ for the vector containing the entries of the matrix $\tilde{\mathtt{P}}$. A solution of $\mathtt{A}\mathbf{p} = \mathbf{0}$, subject to $\|\mathbf{p}\| = 1$, is obtained from the unit singular vector of A corresponding to the smallest singular value.

(ii) **Minimize geometric error.** Using the linear estimate as a starting point minimize the geometric error (7.4):

$$\sum_i d(\tilde{\mathbf{x}}_i, \tilde{\mathtt{P}}\tilde{\mathbf{X}}_i)^2$$

over $\tilde{\mathtt{P}}$, using an iterative algorithm such as Levenberg–Marquardt.

(iii) **Denormalization.** The camera matrix for the original (unnormalized) coordinates is obtained from $\tilde{\mathtt{P}}$ as

$$\mathtt{P} = \mathtt{T}^{-1}\tilde{\mathtt{P}}\mathtt{U}.$$

Algorithm 7.1. *The Gold Standard algorithm for estimating* P *from world to image point correspondences in the case that the world points are very accurately known.*