A LINEAR PREFILTER FOR IMAGE SAMPLING WITH RINGING ARTIFACT CONTROL

G.Blanchet [1,2], *L.Moisan* [1,2] , *B.Rougé* [2,3]

[1] MAP5, Université Paris 5, France
[2] CMLA, École Normale Supérieure de Cachan, France
[3] Centre National d'Études Spatiales, Toulouse, France
{ blanchet, moisan, rouge }@cmla.ens-cachan.fr

ABSTRACT

When sampling a continuous image or subsampling a discrete image, aliasing artifacts can be controlled by filtering the data prior to sampling. Bandlimiting filters completely avoid aliasing artifacts, but have to find a compromise between blur and ringing artifacts. In this paper, we propose a new joint definition of blur/ringing artifacts, that associates to a given bandlimited prefilter its so-called *Spread-Ringing curve*. We then build a set of filters yielding the optimal blur/ringing compromise according to the previous definition. We show on experiments that such filters yield sharper images for a given level of ringing artifact.

1. INTRODUCTION

Producing a high quality low-resolution image from a high-resolution one, or building a high quality image from a continuous model both require a good understanding of the sampling process. The solution to avoid aliasing artifacts, that may introduce dramatic texture changes, losses of connectedness or staircase effects, has been given in 1949 by Shannon [1]: aliasing can be avoided by bandlimiting the input image (that is, setting to 0 its high frequency components), or, equivalently, by filtering the image with a *sinc* filter. Unfortunately this filter oscillates with slow decay, and introduces unwanted oscillations in the vicinity of sharp edges. Since then, other filters were developed such as splines [2], prolate [3] or Cesàro filter [4] but the necessary trade-off between blur and ringing they introduce was never really evaluated.

In the literature, the ringing artifact (sometimes referred to –unproperly– as the Gibbs phenomenon [5]) is often described as noise and is measured with the SNR [6]. More specific measures of ringing, like the perceptual ringing metric [7] and the visible ringing metric [8], manage to control the amount of ringing artifacts put in a signal, but do not try to define the ringing phenomenon by itself.

In this paper, we propose a joint definition of ringing and blur artifacts, yielding what we call a *Spread-Ringing curve* (Section 2). This curve allows us to compare the relative sharpness of classical bandlimited prefilter for any given level of ringing (see Fig. 5). In Section 3 we numerically design an optimal filter (*Spread-Ringing* filter (SR)) that minimizes both ringing and blur artifacts, and compare it with other filters (Fig. 5). By preserving better edges, this filter reduces the deterioration of image quality usually observed with classical filters, as shown by the image experiment reported on Fig. 3 and 4.

2. THE SPREAD-RINGING CURVE

In all the following, we shall only consider 1D signals and filters. Associated 2D filters for images will be built in a separable way. Let us first define the set of bandlimited signals by

$$B_W = \{g \in L^2(\mathbb{R}),\ \mathrm{supp}\,(\hat{g}) \subset [-W, W]\},$$

$\hat{g}$ standing for the Fourier Transform of g. The ringing artifact may occur when a signal f is projected onto B_W (frequency cutoff), or more generally when it is *approximated* by a signal $g \in B_W$. We propose the following definition.

Definition 1 *When we approximate a signal $f \in L^2(\mathbb{R})$ by a bandlimited signal $g \in B_W$, g may have additional oscillations the sampled signal $f(k\pi/W)$ does not have: this is the **continuous ringing phenomenon**.*

Note that the *discrete ringing phenomenon* could be defined as well by considering only oscillations of the sampled signal $g(k\pi/W)$. Now following Definition 1, we would like to define a way to *measure* ringing artifacts, in order to be able to constrain them to a non-perceptible level (that is, below the image noise level).

Let us consider a filter $\varphi \in B_W$, and apply it to the Heaviside function (H), yielding a new signal $g = H * \varphi$. Two phenomena may appear for g : first, the transition from 0 to 1 has a certain spread; second, oscillations may appear around this transition. Defining a measure only for these oscillations (ringing) is difficult because in general there is

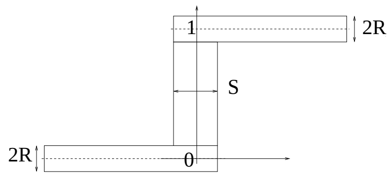

Fig. 1. $D_{S,R}$ domain.

no natural way to define the transition domain (spread) of g. This is why we choose to simultaneously measure the spread of the transition (S) and the amplitude of the oscillations outside the transition domain (R) by constraining the graph of g, $\Gamma_g = \{(x, g(x));\ x \in \mathbb{R}\}$ to be contained in a certain domain (Fig. 1).

Definition 2 *We denote $D_{S,R}$ the subset of $\mathbb{R}^2$ defined by*

$$D_{S,R} = \left\{ \ (x,y) \ \middle| \ \begin{array}{ccc} x \le -\frac{S}{2} & \textit{and} & |y| \le R \\ \textit{or } |x| \le \frac{S}{2} & \textit{and} & -R \le y \le 1+R \\ \textit{or } x \ge \frac{S}{2} & \textit{and} & |1-y| \le R \end{array} \right\}.$$

We observe that a small R prevents too large oscillations of g, while a small S ensures a sharp edge approximation of the discontinuity of H. Hence, the couple (S, R) reflects the blur/ringing trade-off to be satisfied by g. This leads to

Definition 3 *The Spread-Ringing domain associated to a filter $\varphi \in B_W$ is*

$$D(\varphi) = \{(S,R) \in (0,+\infty);\ \ \Gamma_{\varphi * H} \subset D_{S,R}\}. \quad (1)$$

The Spread-Ringing curve associated to φ is the boundary of $D(\varphi)$.

If $(S, R) \in D(\varphi)$, so does $(S + p, R + q)$. So the boundary of $D(\varphi)$ is obtained by taking the minimal possible value of R for any fixed S, or the minimal value of S for a fixed R. The Spread-Ringing curve can be described by the graph of a function

$$r_\varphi(S) = \min\{R \in (0,+\infty); (S,R) \in D(\varphi)\}.$$

This definition of ringing has the drawback that no unique value of S and R are associated to a given filter since, as we mentioned before, it is difficult to distinguish the edge from the ringing near the transition. However, this construction has the advantage to remain very general, since it does not rely on any arbitrary threshold. Moreover, Definition 3 can be generalized to a *family* of filters φ_α (α being in general a real parameter) and still yields a single *Spread-Ringing curve*, as specified in

Definition 4 *The Spread-Ringing domain associated to a family of filters $(\varphi_\alpha)_{\alpha \in A}$ is*

$$D\left((\varphi_\alpha)_{\alpha\in A}\right) \ = \ \bigcup_{\alpha \in A} D(\varphi_\alpha). \quad (2)$$

Its Spread-Ringing curve is the associated boundary.

3. SR FILTERS

We now build a family of bandlimited filters (φ_S), called *SR filters*, having the best possible *Spread-ringing curve*. These filters have minimal ringing R for a fixed spread S, or equivalently, minimal spread S for a fixed ringing R. For convexity reasons, this family exists and is unique. For each S, φ_S can be computed by the following iterative algorithm:

set $\varphi = \delta_0$ (Dirac)
assign a large value to R
repeat
  set $\varphi_S = \varphi$
  set $N = 1$
  repeat
    force $\Gamma_{\varphi * H} \subset D_{S,R}$ by thresholding
    force $\varphi \in B_W$, that is $\hat\varphi(\xi) = 0$ for $\xi \notin [-W, W]$
    set $N = N + 1$
  until *convergence test* or $N > N_{max}$
  reduce R
until $N > N_{max}$

The *convergence test* is satisfied when the two forcing steps have small enough effects on φ, that is, when the first forcing step changes φ by less than ε_1 (according to L^∞ norm) and the second forcing step changes φ by less than ε_2 (according to the L^2 norm). In practice, we checked that convergence was undoubtedly attained with $N_{\text{max}} = 10000$ and $\varepsilon_1 = \varepsilon_2 = 0.001$.

Note that the value of W is arbitrary since different values of W simply mean different scales. Numerically, we used 1024 samples to represent φ and chose a frequency cutoff corresponding to a reduction by a factor 16.

When looking at SR filters (obtained with this algorithm), we noticed that their Fourier transform was real symmetric (as expected), but not unimodal (that is, not both increasing on $\mathbb{R}^-$ and decreasing on $\mathbb{R}^+$), due to large weights on the frequencies near W (and $-W$). This phenomenon, due to the fact that the ringing is controlled through a L^∞ norm, can be undesired in some applications (since for example, natural images generally have unimodal spectra). It can be avoided by building *unimodal SR filters* (see Fig. 2), obtained by adding a third forcing step in the algorithm (between the $D_{S,R}$ forcing step and the B_W one) which changes $\hat\varphi$ into its L^2 unimodal regression.

Remark: We chose to control the ringing with the L^∞ norm (instead of L^1 or L^2 norms) because our perception of images is more sensitive to large local overshoots than to small oscillations (hidden by image noise or textures) spread on a large domain. In that sense, the characterization of ringing we proposed is specific to images and may not be adequate in a general signal processing framework.

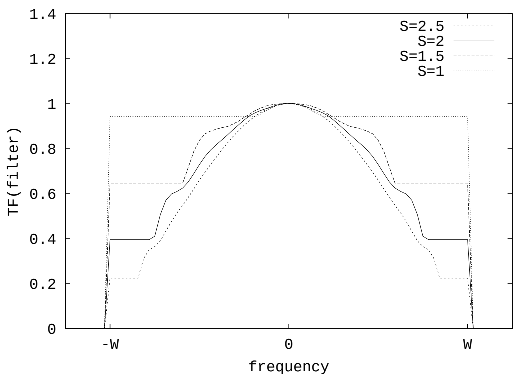

Fig. 2. Some unimodal SR filters in Fourier domain

4. COMPARISON WITH CLASSICAL FILTERS

We compare the results of the SR filters with the following classical filters:

- *sinc* filter: $\forall \xi \in \mathbb{R},\ \widehat{\mathrm{sinc}}(\xi) = 1_{[-W,W]}(\xi)$.
- Frequency truncated triangle ($\alpha \in [0, 1]$):

$$\widehat{T_\alpha}(\xi) = \left(1 + \frac{\alpha - 1}{W}|\xi|\right) \cdot 1_{[-W,W]}(\xi)$$

- Cosine filter ($\alpha \in \mathbb{R}$):

$$\widehat{C_\alpha}(\xi) = \frac{1 + \cos(\alpha\xi)}{2} \cdot 1_{[-W,W]}(\xi)$$

- Prolate [3] and bandlimited Gaussian filters.

As expected, SR filters (Fig. 5) show better results than other tested filters. They introduce less ringing for any given edge spread. The unimodal constraint reduces the performance of the filter, as the *Spread-Ringing curve* of the unimodal filter (a) is on the right hand side of the *Spread-Ringing curve* of the non unimodal filter (b). This divergence gets magnified for small spread values.

Among other filters, we observe two groups. The first group (c,d,e) contains filters that give relatively good results in comparison with the SR filter. For a spread value smaller than 2 pixels (which is a common level of sharpness), they give very similar results, especially the prolate (c) and the cosine filter (d). The filters of the second group (f,g) perform poorly as they produce a high level of ringing.

5. EXPERIMENTS

We now apply SR filters to a natural image. We generalize the 1D filter to 2D by using a separable convolution along both coordinates. Fig. 3 shows results obtained on a subpart (512×512) of a natural image. In this experiment,

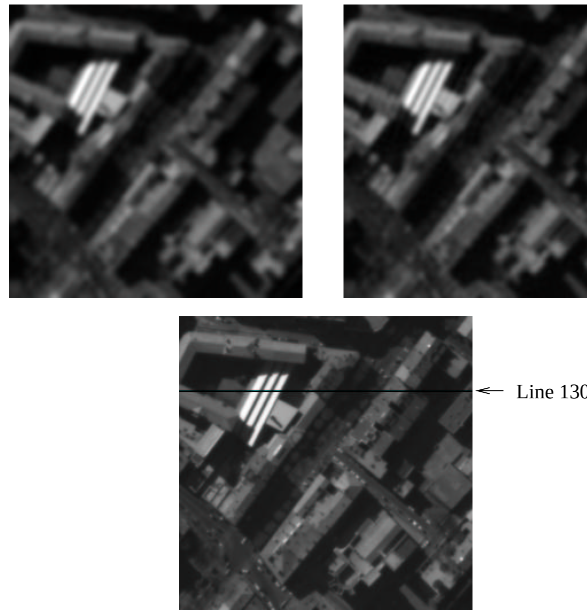

Fig. 3. The reference high-resolution (not bandlimited) town image I (bottom row) is prefiltered with the prolate filter (yielding the top left image: I_P) and with the SR filter (top right image: I_{SR}). One can see that the white roof is better preserved with the SR filter (transitions look sharper).

we chose the SR filter corresponding to $R = 0.012$, allowing 1.2% of overshoot with respect to the transition value, which approximately corresponds to the level of noise of the image. The convolution of the original high resolution image (bottom of Fig. 3) was made with this SR filter and with a prolate filter (with same value of R) as a comparison. As predicted by the SR curves, the prolate filter yields a more blurry image, as shown on Fig. 3 and 4.

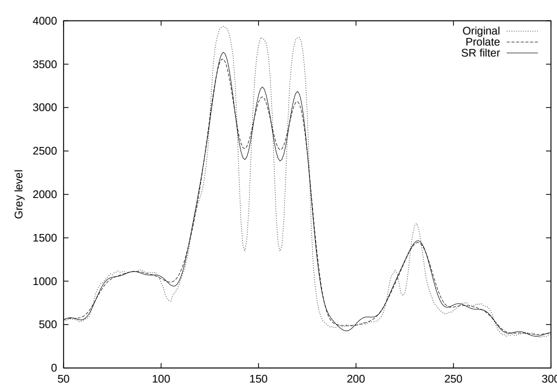

Fig. 4. Line 130 from the images of Fig. 3 near the white roof. The amplitude of the transitions are better preserved with the SR filter than with the prolate. The difference between I and both I_P and I_{SR} confirms the improvement: $||I - I_P||_1 = 90$, $||I - I_{SR}||_1 = 83$, $||I - I_P||_2 = 150$, $||I - I_{SR}||_2 = 137$.

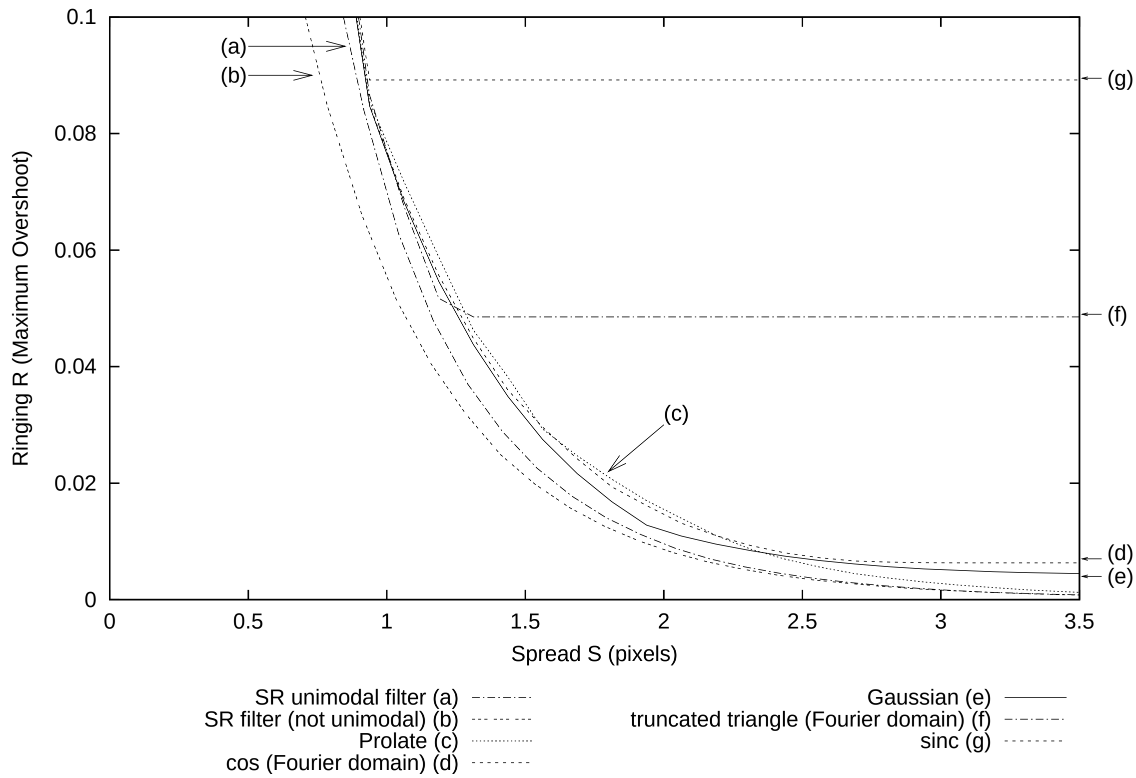

Fig. 5. *Spread-Ringing curves* of some bandlimited low-pass filters. These curves display the trade-off between ringing and blur artifacts for each filter. This representation incidentally shows that bandlimited Gaussian filters perform slightly better than prolate filters for small spreads ($S < 2.5$ pixels), which are the most useful values in practice. As expected, SR filters achieve the best compromise (more than 20% better than the others for $S = 1.5$ pixels).

6. CONCLUSION

We introduced a joint measure of blur and ringing artifacts for bandlimiting filters, and built SR filters minimizing these artifacts. Compared to classical filters, SR filters yield perceptible improvements when bandlimiting an image before sampling, which could be interesting for applications requiring high quality image reduction. This approach also pushes back the frontier of attainable aliasing/blur/ringing compromises in image formation processes. In this paper, 2D filters are built in a separable way, but generalization to non-separable filters (radially symmetric or not) could be investigated as well.

7. REFERENCES

[1] C. E. Shannon, "Communications in the presence of noise", *Proc.of the IRE*, vol. 37, pp. 10-21, 1949.

[2] P. Thévenaz, T. Blu, M. Unser , "Image Interpolation and Resampling", *Handbook of Medical Imaging, Processing and Analysis*, I.N. Bankman Ed., Academic Press, pp. 393-420, 2000.

[3] D. Slepian, "Prolate Spheroidal Wave functions, Fourier Analysis, and Uncertainty- V: The Discrete Case", *Bell System Technical Journal*, 1978.

[4] A. J. Jerri, *The Gibbs Phenomenon in Fourier Analysis, Splines and Wavelet Approximations*, Kluwer Academic, 1998.

[5] E. Hewitt, R. Hewitt, "The Gibbs-Wilbraham phenomenon: An episode in Fourier Analysis", *Hist. Exact Sci.*, pp. 129-160, 1979 .

[6] A. Muñoz, T. Blu, M. Unser "l_p-Multiresolution Analysis: How to Reduce Ringing and Sparsify the Error", *IEEE Trans. Image Process.*, vol. 11:6, 2002.

[7] P. Marziliano, F. Dufaux, S. Winkler, T. Ebrahimi, "Perceptual Blur and Ringing Metrics: Application to JPEG2000", *SPIC*, vol. 19, pp. 163-172, 2004.

[8] S.H. Oguz, *Morphological post-filtering of ringing and lost data concealment in generalized lapped orthogonal transform based image and video coding*, Ph.D dissertation, Univ of Wisconsin, Madison, 1999.