Determinantal processes, random matrices and hyperuniformity

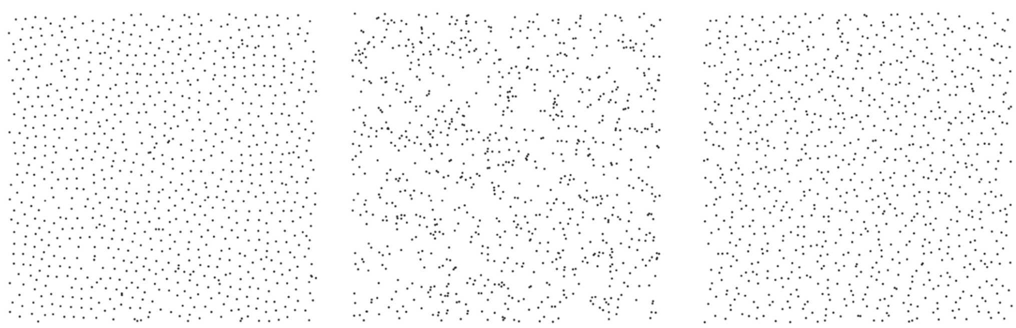
R. Lachièze-Rey

Course Description

The purpose of this course is to study random configurations of points (or particles) in continuous space $\mathbb{R}^d$, called **point processes**. We begin by studying finite systems, starting with a set of N independent and identically distributed points in a compact set of $\mathbb{R}^d$, and their natural generalization to an infinite number of points, the *Poisson process*, which serve as a "reference law" (Figure below).

The N eigenvalues of $N \times N$ real or complex **random matrices** provide models of highly dependent points in $\mathbb{R}$ or $\mathbb{C}$, asymptotically related to some **determinantal point processes**, particularly the "sine process" (translation invariant on $\mathbb{R}$) or the "infinite" Ginibre process (translation invariant on $\mathbb{C}$). Determinantal processes form a very rich class of models in any dimension, but these two experience a variance cancellation phenomenon that gives them particular macroscopic properties. This phenomenon is called **hyperuniformity**.

Depending on time, we may study other systems with this particular property, such as the zeros of the planar **Gaussian analytic function**, related to the study of zeros of random polynomials (this would require some complex analysis), or certain **continuous particle systems**, or study what hyperuniformity implies macroscopically (optimal transport, rigidity, etc...).



Left: Zeros of a random function. *Middle:* Independent "Poisson" points. *Right:* Eigenvalues of a (large) random matrix

Keywords: Point processes, random matrices, determinantal processes, hyperuniformity, zeros of random polynomials, Particle Systems, Gibbs measures, Optimal Transport.

References

[BKPV] Zeros of Gaussian Analytic Functions and Determinantal Point Processes, John Ben Hough, Manjunath Krishnapur, Yuval Peres, Bálint Virág.