Optimal estimation of the derivatives of a density

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Abstract

Let $X_1, \dots, X_n$ be independent and identically distributed random variables, with common density denoted by f. In this work, we are interested in the estimation of $f^{(d)}$ the d-th order derivative of f. We present a new procedure of nonparametric estimation of the density derivatives $f^{(d)}$ on $\mathbb{R}$ or $\mathbb{R}^+$, based on a projection method using the Hermite basis or the Laguerre basis, see [1], [2]. We study the risk of our estimator, proving upper bound on the $\mathbb{L}^2$-integrated risk on $\mathbb{L}^2(\mathbb{R})$ or $\mathbb{L}^2(\mathbb{R}+)$. Then, we derive consistency of our estimator of $f^{(d)}$ and prove that its rate has order $n^{-2(s-d)/2s+1}$ if the density belongs to Sobolev-Hermite or Sobolev-Laguerre ball, associated with regularity order s, $s > d$. A lower bound is also proved, thus the rate $n^{-2(s-d)/2s+1}$ is optimal in the sense minimax. We propose a data driven procedure: the resulting estimator realizes automatically a bias-variance compromise. We compare our results to [3], [4]. Lastly, the method is practically illustrated through numerical experiments.

References

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