HW1 - Algebra - Fall 2018

Due date: Friday, September 21.

1 Binary operations

Let S be a finite set, and let $\bullet$ be a binary operation on S which is associative, and for which there exists a neutral element e. Let s in S be an element that has a "left-inverse", i.e.

$$\exists t \in S, \quad t \bullet s = e.$$

We want to prove that t is also the "right-inverse" i.e. $s \bullet t = e$.

1. Consider the map $\varphi : S \to S$ defined by

   $$\varphi(r) := s \bullet r.$$

   Show that it is one-to-one. (Hint: use the fact that s has a left inverse!!)

2. Since S is finite, what can you deduce about the map φ?

3. Conclude that there exists a right-inverse for s, i.e. there exists u in S such that $s \bullet u = e$.

4. Show that $t = u$ (use associativity).

Optional Can you find a counter-example if S is not finite?

2 Groups

Exercises 2, 7, 27, 31 (chapter 3).

3 Subgroups

Exercises 41, 45 (chapter 3).

4 Historical extra-credit

In a couple of sentences, what do you think of the life of Évariste Galois?