Algebra – HW 3 – Solutions

Ex. 3 chap 5

a) Even b) Even × Even = Even c) odd × even = odd

d) even × odd × odd = even e) odd

Ex. 20 We argue by induction

[$n=1$ there is only the identity (a "cycle of length 1") which is the product of zero transposition.]

$n=2$ only cycles $\leftrightarrow$ transpositions themselves

Assume it is true for n. Let C be a cycle in S_{n+1}.

- If C is a cycle of length $\leqslant n$, then it can be seen as a cycle in S_n and we apply induction hypothesis.
- If C is a cycle of length $n+1$, we write it as

$$C = (c_1 \ldots c_{n+1}) = \underbrace{(c_1 \cdots c_n)} (c_n\, c_{n+1})$$

By induction, this can be written as a product of at most $n-1$ transpositions,

so C is the product of at most n transpositions.

Ex. 24 A 3-cycle $(a_1\, a_2\, a_3)$ can be written as

$(a_1 a_2 a_3) = (a_1\, a_2)(a_2\, a_3)$ So it is even

Ex. 27 We only have to show that λ_g is bijective (that is the definition...)

Injective? $ga = gb \Rightarrow a = b$ by applying g^{-1}

Surjective $ga = b \iff a = g^{-1}b$ //

Remark: The question should be stated as "let G... and let $g \in G$, show that..."

34 $\varepsilon(\alpha\alpha^{-1}) = \varepsilon(id) = 1$

$= \varepsilon(\alpha)\,\varepsilon(\alpha^{-1})$ so if $\begin{cases} \varepsilon(\alpha) = 1 \text{ then } \varepsilon(\alpha^{-1}) = 1 \\ \varepsilon(\alpha) = -1 \text{ then } \varepsilon(\alpha^{-1}) = -1 \end{cases}$.