

## HW9 - Algebra - Fall 2018

**Due date:** Friday, Dec 14th.

1. Show that  $\sqrt{6}$  cannot be written as

$$\sqrt{6} = a + b\sqrt{2} + c\sqrt{3},$$

with  $a, b, c$  rational numbers.

2. Let  $F$  be a field,  $P$  be a polynomial in  $F[X]$  and let  $I$  be the ideal generated by  $P$ . Prove the equivalence between
  - $P$  is irreducible
  - $I$  is prime
  - $I$  is maximal.
3. Let  $E, F$  be two fields. Write the definition of a “field morphism” between  $E$  and  $F$ .
4. Show that the only field morphism from  $\mathbb{Q}$  to itself is the trivial one, namely the identity map.
5. Show that there is a non-trivial isomorphism from  $\mathbb{Q}(\sqrt{2})$  to itself.