Algebra - syllabus for final exam

Group, subgroup, trivial subgroup, Abelian group. The groups $\mathbb{Z}$ and $\mathbb{Z}_n$ for $n \geq 2$. Subgroup generated by an element. Cyclic group. Order of a group, order of an element. Group morphism, isomorphism, kernel. Group S_n of permutations on n elements, transposition, cycle, parity/signature. Direct product of two groups. Normal subgroup.

Not needed: group actions, Cayley's and Lagrange's theorem.

How to prove that something is a subgroup. How to prove that a group is cyclic or not. How to compute the order of an element. How to check that something is a group morphism. How to show that two groups are isomorphic, or not. The signature/parity is a group morphism. How to compute the parity of a permutation.

The kernel of a group morphism is a (normal) subgroup. Every subgroup of a cyclic group is cyclic. If a group is Abelian (resp. cyclic) then every group isomorphic to it is also Abelian (resp. cyclic). Every permutation can be written as a product of cycles with disjoint support, and as a product of transpositions.

Ring, subring, commutative ring, the unity in a ring. Ring morphism, kernel of a ring morphism. Ideals in a commutative ring, trivial ideals. Principal ideal, prime ideal, maximal ideal. Direct product of rings. Quotient ring.

Not needed: the actual construction of the quotient ring. But you should know how to "use it".

How to prove that something is a subring, an ideal. How to prove that an ideal is prime, maximal, principal.

The kernel of a ring morphism is an ideal. Every ideal in $\mathbb{Z}$ is principal. If an ideal contains the unity, it is trivial. In a commutative ring, every maximal ideal is prime.

Not needed: the ideals of $R \times S$ are product of ideals.

Integral domain, divisors of zero. R/I is an integral domain if and only if I is prime.

Field, subfield, extension field. How to prove that something is a subfield.

Not needed: field of fractions, Kronecker's theorem.

The ideals of a field are trivial.

Polynomial in $F[X]$, the "division algorithm", every ideal of $F[X]$ is principal, irreducible polynomials.