

1. In Exercise 1, Q.4, the question had to be corrected in some way, as mentioned during the exam, because if J is included in I , then $I + J = I$.
2. In Exercise 2, Q.1. The question, as stated, is correct, and a Boolean ring is in fact **both** commutative and anti-commutative. It is slightly easier to show (as indicated during the exam) that R is anti-commutative, by expanding

$$a + b = (a + b)^2 = a^2 + b^2 + ba + ab = a + b + ba + ab,$$

which implies $ab = -ba$.

In fact, we can also prove that on the other hand $c = -c$ for all $c \in R$, so the ring is also commutative.

3. Exercise 4, Q.3, was wrong (my apologies). It should read: write down a non-trivial ideal of $\mathbb{Z}[\sqrt{2}]$, instead of $\mathbb{Q}[\sqrt{2}]$.

There is no non-trivial ideal in $\mathbb{Q}[\sqrt{2}]$ because every non-zero element is invertible (for the product) in $\mathbb{Q}[\sqrt{2}]$ (it is a field).

Everyone gets the 5 points on this question.

4. Exercise 4, Q.4 had to be corrected as mentioned at the beginning, because the ideal is not prime. For example $\sqrt{2} \times \sqrt{2} = 2$ is in the ideal, but $\sqrt{2}$ is not.