Mathematical Statistics - Section 1 - NYU Spring 2019 - Homework 10

Causality

We recall that we encode the "type" T of an individual as a function from $\{0,1\}$ to $\{0,1\}$, and that we define the causal effect θ as

$$\theta := \mathbb{E}[T(1)] - \mathbb{E}[T(0)].$$

If you prefer, you can encode T as a vector of size 2, whose first coefficient is the value $T(0)$ and the second coefficient is the value $T(1)$.

1. Give an example of a distribution on T such that $\theta > 0$, and one such that $\theta = 0$.

If (X, T) is a random vector where X is a binary variable in $\{0,1\}$ and T is a "type", we define the association effect α as

$$\alpha := \mathbb{E}[T(1)|X = 1] - \mathbb{E}[T(0)|X = 0].$$

The marginal distribution of X must be a Bernoulli with parameter $1/2$.

2. Give an example of a distribution on (X, T) where $\theta > 0$ but $\alpha < 0$.

3. Give an example of a distribution on (X, T) where $\theta = 0$ but $\alpha > 0$.

Let Z be a random variable of your choice (it could e.g. be another binary random variable).

4. Give a distribution on (X, T, Z) such that T and X are not independent, but T and $X|Z$ are independent (i.e. T and X are independent for a fixed value of Z).

We recall that there are four possible Types, to which you can give names if needed.

To define distributions, one option (among others) is to simply declare

$$(X,T) = \begin{cases} (0, Type_1) & \textit{with probability ***}, \\ (0, Type_2) & \textit{with probability ***}, \\ \textit{etc.} \end{cases}$$

Of course, not all possibilities need to be used with positive probability.