

HW 10 - Solution

Types: $T_1: \begin{matrix} 0 \rightarrow 0 \\ 1 \rightarrow 0 \end{matrix}$ ("Sleepy") $T_3: \begin{matrix} 0 \rightarrow 1 \\ 1 \rightarrow 1 \end{matrix}$ ("Wide")
 $T_2: \begin{matrix} 0 \rightarrow 0 \\ 1 \rightarrow 1 \end{matrix}$ ("Mainstream") $T_4: \begin{matrix} 0 \rightarrow 1 \\ 1 \rightarrow 0 \end{matrix}$ ("Allergic")

1) If everyone has type T_2 , then

$$\Theta = E[T_2(1)] - E[T_2(0)] = 1 > 0$$

If $\frac{1}{2}$ of the pop. is T_2 and $\frac{1}{2}$ is T_4

or if everyone is T_1 , or if everyone is T_3 , or ...

then $\Theta = 0$.

2) Take 80% of sleepy, 15% of mainstream, 5% of allergic

$$\Theta = 0.15 - 0.05 = 0.1 > 0$$

~~Give X to the allergic, and to the sleepy until we reach 50%.~~

~~Don't give X to the mainstream, and to the rest of the sleepy ones.~~

~~$$\alpha = E[T(1) | X=1] - E[T(0) | X=0]$$~~

Give X to 50% among the 80% sleepy

not X to 30% sleepy; 15% mainstream; 5% allergic.

$$E[T(1) | X=1] = 0$$

$$\text{so } \alpha = -0.1 \leq 0$$

$$E[T(0) | X=0] = 0.1$$

3) Take T_1, T_2, T_3, T_4 25% each, so $\Theta = 0$.

Give X to ~~3~~ Mainstream Woke not X to Sleepy Allergic

$$E[T(1) | X=1] = 1; \quad E[T(0) | X=0] = 0.5$$

So $\alpha > 0$.

4) A simple case: take $(X, T, Z) = (1, T_1, 1)$ 50%
 $(X, T, Z) = (0, T_2, 0)$ 50%.

Then X and T are not \perp because knowing $X \rightarrow$ knowing T
but for Z fixed; X and T are constant... hence \perp !