**Mathematical Statistics - Section 1 - NYU Spring 2019 - Homework 4**

**MLE for Poisson** We recall that, for $\lambda > 0$, the Poisson distribution $\pi_\lambda$ with parameter $\lambda$ is a distribution on the integers $k \geq 0$ given by

$$\pi_\lambda(X = k) := e^{-\lambda} \frac{\lambda^k}{k!}$$

1. Compute the maximum likelihood estimator for the Poisson distribution.

**The uniform distribution** Here we choose for statistical model the family of uniform distributions $\{U_\theta\}_\theta$, where the pdf of $U_\theta$ is given by

$$U_\theta(x) = \begin{cases} \frac{1}{\theta} & \text{if } x \in [0, \theta] \\ 0 & \text{otherwise.} \end{cases}$$

We let $\theta_\star$ be the "real" parameter, that we try to infer.

2. Compute the first and second moment of $U_\theta$. Deduce two estimators of $\theta_\star$ based on the method of moments. *Hint: we mentioned those in class.*

3. Compute the variance of the estimator based on the first moment *Hint: this should be almost like computing the variance of the empirical mean.*

4. We recall that the MLE in this case is given by $\widehat{\theta}_n := \max_{i=1,\dots,n} X_i$. Compute the probability distribution function of $\widehat{\theta}_n$.

   *Hint: compute* $\frac{1}{\delta}\mathbb{P}\left(\max_{i=1,\dots,n} X_i \in [x, x+\delta]\right)$ *for* $\delta > 0$ *arbitrarily small.*

5. Compute the expectation of $\widehat{\theta}_n$.

6. Compute the variance of $\widehat{\theta}_n$, and compare it to the result found in question 3.