

# HW6 - Solution

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5)

a) Power function  $\beta(\theta) = P_{\theta}(Y > c)$   
 $= P_{\theta}(\max\{X_1, \dots, X_n\} > c)$   
 $= 1 - P_{\theta}(\max \leq c) = \begin{cases} 1 - \left(\frac{c}{\theta}\right)^n & \text{if } c \leq \theta \\ 0 & \text{if } c \geq \theta \end{cases}$

b) We have size  $= \beta\left(\frac{1}{2}\right) = \begin{cases} 1 - (2c)^n & c \leq \frac{1}{2} \\ 0 & c \geq \frac{1}{2} \end{cases}$   
size 0.05

$\Leftrightarrow 1 - (2c)^n = 0.05 \Leftrightarrow \boxed{\frac{(0.95)^{1/n}}{2} = c}$

c)  $1 - (2 \times 0.48)^{20} = 1 - (0.96)^{20} \approx 1 - 0.44 \approx \boxed{0.56}$   
p-value.

I would keep  $H_0$

d) p-value = 0. Reject  $H_0$  with confidence, even ~~certainty~~ certainty!

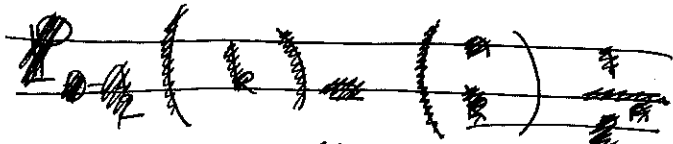
Rem. This is not a great test. If  $\theta = \frac{1}{2}$ , the max should be around  $\frac{1}{2}$ , and we should reject if it is too small.

6]  $\theta$  = probability of dying before rather than after.

Belief  $\theta = \frac{1}{2}$

$n = 1919$

$k = 922$  \* deaths before rather than after.



Pearson's  $\chi^2$ -statistic  $T = \frac{(922 - \frac{1919}{2})^2}{\frac{1919}{2}} + \frac{(997 - \frac{1919}{2})^2}{\frac{1919}{2}}$

$= \frac{1406.25}{959.5} + \frac{1406.25}{959.5}$

$= 2.93$   
should be a  $\chi^2$ -distribution

p-value  $= P_{\chi^2}(X \geq 2.93)$  between 5% and 10% ~~Reject~~ Reject  $H_0$

Confidence interval? Can use empirical mean.

$\hat{\theta} = \frac{922}{1919} = 0.48$   $n = 1919$

This is way too good to be true

$(\hat{\theta} - \theta) \frac{\sqrt{n}}{\hat{\theta}(1-\hat{\theta})} \approx N(0,1)$

$\approx \frac{3}{10000}$

$|\theta - \hat{\theta}| \leq \frac{c \hat{\theta}(1-\hat{\theta})}{\sqrt{n}}$  with proba  $\approx P(N(0,1) \geq c)$

$|\theta - \hat{\theta}| \leq \frac{c}{4}$

$\theta \in [0.48 - \frac{1.4}{4 \times 959.5}, 0.48 + \frac{1.4}{4 \times 959.5}]$

Should be 2 instead of 4  $4 \times 959.5$

959.5 is  $(1919)/2$ , whereas it should be  $0.48 + \frac{1.4}{4 \times 959.5}$

$c = 1.4$   $(1919)^{(1/2)}$  with ~~90~~ 95% confidence

7) a) Compute  $\hat{\mu}_1, \hat{\mu}_2$ , and  $\hat{\mu}_1 - \hat{\mu}_2$  (difference of empirical means)  
Then compute standard errors.

If we consider that  $n$  is a large number, we can perform Wald's test and check whether  $P(|N(0,1)| \geq \frac{\hat{\mu}_1 - \hat{\mu}_2}{\sqrt{\frac{se_1^2}{n} + \frac{se_2^2}{n}}})$  is small or not...

8) The distribution of  $T$  is  $N(\theta, \frac{1}{n}) = \theta + N(0, \frac{1}{n}) \cdot \frac{1}{\sqrt{n}}$

a) so  $P_{\theta=0}(T > c) = P_{\theta=0}(\theta + \frac{1}{\sqrt{n}} N(0,1) > c)$

$= P(N(0,1) > c\sqrt{n})$

if we want  $\leq \alpha$ , take  $z(\alpha)$  s.t.  $P(N(0,1) \geq z) \leq \alpha$

and let  $c = \frac{z(\alpha)}{\sqrt{n}}$

b)  $\beta(1) = P_{\theta=1}(1 + \frac{1}{\sqrt{n}} N(0,1) > c)$

c) as  $n \rightarrow \infty$ , since  $c \rightarrow 0$  we have indeed

$\beta(1) \xrightarrow{n \rightarrow \infty} P_{\theta=1}(1 > 0) = 1$

9) By asymptotic normality of MLE,

$\frac{\hat{\theta} - \theta_0}{\hat{se}} \rightarrow N(0, 1)$

and thus  $\beta(\theta) \rightarrow 1$

so  $\frac{\hat{\theta} - \theta_0}{\hat{se}} = \frac{\hat{\theta} - \theta}{\hat{se}} + \frac{\theta - \theta_0}{\hat{se}}$   
 $\rightarrow N(0,1) \quad \rightarrow +\infty$

