Mathematical Statistics, Final Exam, due May 9

★**1.** True or false? Give arguments.

(a) If two different estimators are unbiased, they are equally as good and it does not matter which one we use.

(b) An increased confidence level typically leads to a longer confidence interval.

(c) An increased sample size typically leads to a longer confidence interval.

(d) An opinion poll where the estimated proportion is 90% has a larger margin of error than one where the estimated proportion is 50%.

(e) If we reject a null hypothesis on the 5% significance level, the probability that the null hypothesis is true is less than 0.05.

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★**2.** Let $X_1, X_2, ..., X_n$ be a random sample from a distribution that has pdf $f_\theta(x) = \theta^2 x e^{-\theta x},\ x \geq 0$, where θ is an unknown parameter.

(a) Show that the MLE of θ is $\widehat{\theta} = 2/\bar{X}$.

(b) Suppose that $n = 1$ so that we have only one observation X_1. Is $\widehat{\theta}$ unbiased?

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★**3.** A politician claims to have support from more than 50% of the population. In an opinion poll, 1166 people were asked and 612 of them supported the politician.

(a) Find a 95% symmetric confidence interval for the unknown proportion of supporters p. Does the poll result support the claim?

(b) You can do this as a hypothesis test. State the relevant null hypothesis and alternative hypothesis.

(c) In **(b)**, we reject H_0 in favor of H_A if $T = \dfrac{\widehat{p} - 0.5}{\sqrt{0.5 \cdot 0.5/1166}} > 1.64$. Compute the power of the test if the true value of p is 0.6.

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4. Plastic cups are produced by 2 machines and often come out defective. The defective rates for machines 1 and 2 are 10% and 40%, repsectively. In any given day, machine 1 makes twice as many cups as machine 2, but all cups are put into the same type of containers; thus, a given container contains cups from one of the machines, but we do not know which. A sample of size 4 from a container reveals 1 defective cup and you wish to decide which machine made this sample.

(a) Use the idea of the method of maximum likelihood to decide which machine that produced the cups. Note that there are only two possible parameter values for the unknown proportion p so this is done by trial and error rather than by differentiation.

(b) Use the idea of Bayesian inference to decide which machine that produced the cups. Note that the prior distribution of p has only two possible values so this boils down to the old Bayes' rule with two events.

(c) If you have solved the problem correctly, you get different conclusions in **(a)** and **(b)**. Explain why (specifically what information that can be used in **(b)** but not in **(a)**).

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5. Write one page about the two topics you found most interesting and least interesting in the course. Argue why you found these most/least interesting.

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6. Draw a cartoon that captures the essence of the class.