HW 1 - ODE's - Spring 2018

Due date: Tuesday, January 30^{th}

From order N to order 1 Write the following differential equations (or systems of differential equations) as differential equations of order 1 of the form $Y' = f(t, Y)$ where Y is vector-valued.

1. $y'' + (y')^2 = \cos(t)$
2. $t^2 y' - y\cos(t) + y'' = 1$
3. The system
$$\begin{cases} y_1'' = y_1 + ty_2, \\ y_2'' = y_1 y_2 - y_1' y_2' \end{cases}$$

Comparison of solutions Let $T > 0$ and $f : [0, T] \times \mathbb{R} \to \mathbb{R}$. Let x and y be two differentiable functions on $[0, T]$ such that

$$x'(t) = f(t, x(t)) \quad y'(t) < f(t, y(t))$$

for all t in $[0, T]$. We assume that $y(0) < x(0)$. Show that

$$y(t) < x(t) \quad \text{for all } t \text{ in } [0, T].$$

(We say that y is a *sub-solution* of the differential equation $z' = f(t, z)$.)

Solving Solve the following equations.

1. $(t^2 - 1)x' + 2tx^2 = 0$
2. $2t^2 xx' + x^2 = 2$
3. $x' = 2^{t+x}$

Reading In case you do not feel comfortable with the following notions, do some reading (e.g. Wikipedia). It is not mandatory but it will certainly help understanding the techniques in the next sections.

- What is a Banach space?
- What is the fixed point theorem for contractions of Banach spaces?
- What is the exponential of a matrix?
- What is the Grönwall's inequality (or lemma) about?