HW 9 - ODE's - Spring 2018

Due date: Tuesday, April 17^{th}

A periodic system We consider the system of ODE's

$$\begin{cases} x' &= x + y \\ y' &= y\frac{\cos(t)+\sin(t)}{2+\sin(t)-\cos(t)} \end{cases}$$

1. Write down the matrix $R(2\pi)$ representing the flow at time 2π, in the basis (x, y).
   Hint: first, solve the ODE for y, then plug into the first line and solve for x.
2. Find a matrix P such that $R(2\pi) = e^{2\pi P}$.
3. Find a matrix U such that $R(t) = U(t)e^{tP}$ for all t (Floquet's normal form).

Markus-Yamabe We consider $Y' = A(t)Y$, where A is a π-periodic matrix given by

$$A(t) = \begin{pmatrix} -1 + \frac{3}{2}\cos^2(t) & 1 - \frac{3}{2}\cos(t)\sin(t) \\ -1 - \frac{3}{2}\cos(t)\sin(t) & -1 + \frac{3}{2}\sin^2(t) \end{pmatrix}$$

1. What are the eigenvalues of $A(t)$? Let us call them λ_1, λ_2.
2. Show that
   $$Y(t) = e^{t/2}\begin{pmatrix} \cos(t) \\ -\sin(t) \end{pmatrix}$$
   is a solution to the ODE.
3. Is the solution $Y(t) \equiv 0$ stable?
4. Show that, on the other hand, the solution $Y(t) \equiv 0$ would be stable for the ODE
   $$Y' = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} Y.$$